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GENERATOR OF REACTIVITY FUNCTIONS

REACTORS

KEYWORDS: *reactor kinetics, reactivity coefficients, reactivity worths, control elements, criticality*

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Time-dependent reactivities of many kinds are useful for the analysis of neutron kinetics, such as the study of reactor parameters, neutron wave propagation, or any other specific evolution of the reactor in the time. This is done by imposing the time function directly on a cadmium rod and measuring the neutron flux. Two experiments were performed: In the first one, the reactivity function required for a sinusoidal output was generated with all its parameters in agreement with the previous calculation. In the second experiment, a parameter was slightly altered. Both cases were analyzed, and their results agreed with calculations based on the point reactor model.

INTRODUCTION

Reactivity signal inputs on a nuclear reactor are rather restricted to those that are allowed by both its operational structure and its safety system. Control rods are generally not designed to be manipulated in such a way as to produce a specific output in the neutron flux, or conversely, to analyze the output for a given reactivity function. The basic purpose of the instrument described here is to provide several kinds of reactivity functions of time, either periodical or not, with almost any desired amplitude and time duration, by manipulating the control rod. It is clear that "control rod" means a plate of neutron absorber, i.e., cadmium that can be moved into the core, and need not necessarily be one of those used for the actual reactor control. When high frequencies are involved, the mass of the cadmium plate must be small. Furthermore, once this rod is fitted to the apparatus, it loses its capability as a control or safety device in the usual sense, so that the name "function rod" seems to be more appropriate here.

Two basic precautions must be taken into account when planning the experiment; one is the limit imposed by the greatest reactivity change allowable for safe operation, and the other is the capability for monitoring the experiment from the reactor console during operation.

With the function rod fitted to the apparatus (and with the apparatus being fitted to the reactor), the rod must be calibrated for reactivity versus position to normalize the measurements; however, in cases of very small amplitudes, reactivity can be assumed linear with position. On the contrary, for large amplitudes or nonlinear zones of the core, the function imposed on the rod must be the calibrated one; conversely, if the imposed function is taken on a linear scale, the reactor output will be the one connected with the actual or corrected reactivity function.

As indicated above, monitoring during operation is needed as a precaution, and it should be useful for any eventual input-output analysis. Such monitoring is performed by means of a device that can display the reactivity function, while the reactor output is also displayed by any of the usual methods.

EXPERIMENTAL SETUP

The ρ-generator as a whole is shown in a schematic diagram in Fig. 1. Five basic blocks or components represent the system:

1. electric motor with speed control
2. gearshift box

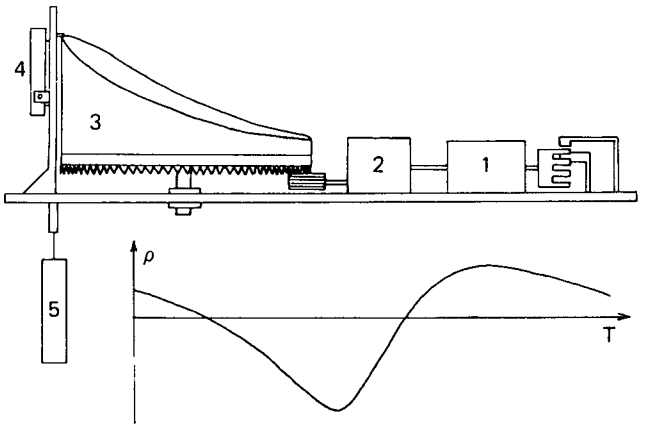


Fig. 1. Generator of reactivities and typical reactivity function for sinusoidal output.

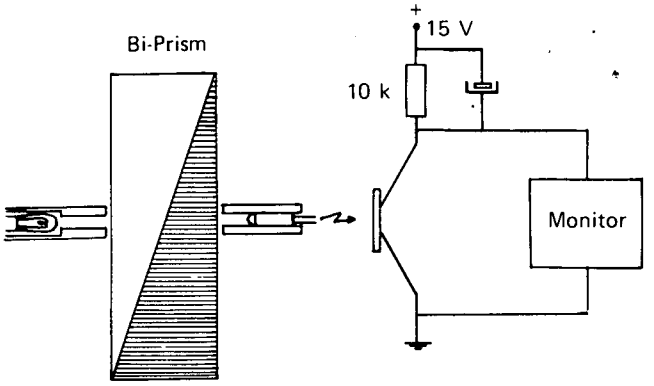


Fig. 2. Bi-prism and monitor system.

3. function crown
4. monitoring instrument
5. function rod.

The first of these components is a direct current (dc) shunt motor, its speed being held constant for the proper range of mechanical torque.

The function of the second component, the gearshift box, is to provide three different speeds, with gear ratios of 1 to 10 to 100, activated by the single motor speed.

The role of the third component is to displace the function rod to obtain the desired reactivity. As can be seen in Fig. 1, it includes a circular plate toothed in the edge of its lower face. The upper face supports the cylindrical crown, in the upper end of which the reactivity function is cut out. This system, both plate and crown, rotates on a vertical spindle thrust inside a roller bearing. At the same time, the plate leans on three other small rollers, each vertically adjustable to level the plate.

The fourth component, the monitoring instrument, consists of two parts: One is a small chariot that follows the vertical movement described by the rotating crown, while supporting the function rod; the second is an acrylic bi-prism that also follows the vertical movement. One of the prisms is neutral tinted or transparent, while the other is lightly colored. As shown in Fig. 2, a beam of light passing through the bi-prism activates a photo-diode that generates an electrical signal whose intensity depends on the vertical position of the bi-prism. Such an arrangement provides the reactivity function display.

The fifth component, the function rod, is a piece of cadmium that can move vertically within the core. Suspended from the chariot, it slides along one of the control rod guides, generating the function traced on the crown. A photographic view of most of the ensemble is shown in Fig. 3.

DISCUSSION

As mentioned above the apparatus is designed for the analysis of both periodic and nonperiodic functions, in as much as they are out of the restrictions imposed by the apparatus itself. In the case of nonperiodic functions, the time operation is inversely proportional to the angular speed of the crown, which can make, as a lower limit, one turn in 100 s, this being, in consequence, the greater time for operation.

Since the function rod, by means of a very small roller, leans on the metallic crown where the function is cut out, care must be taken to avoid separation between roller and crown in the case of a too prompt change of its amplitude. For example, if the function is a sinusoidal one, the condition for no separation is given by

$$W^2 A \sin wt < g \quad , \tag{1}$$

where A is the signal amplitude and g the acceleration due to gravity. From this expression we get the permissible values for the frequencies,

$$\nu < \nu_0 = \frac{1}{2\pi}\left(\frac{g}{A}\right)^{1/2} \quad . \tag{2}$$

For cases in which the rod is immersed in water, extra friction must be taken into account; in this circumstance, the upper limit will be slightly smaller.

Figure 1 shows a periodic but nonsinusoidal function, which is associated with the crown profile depicted in the same figure. This function[1] was calculated to obtain a sinusoidal neutron flux variation for a point model reactor.

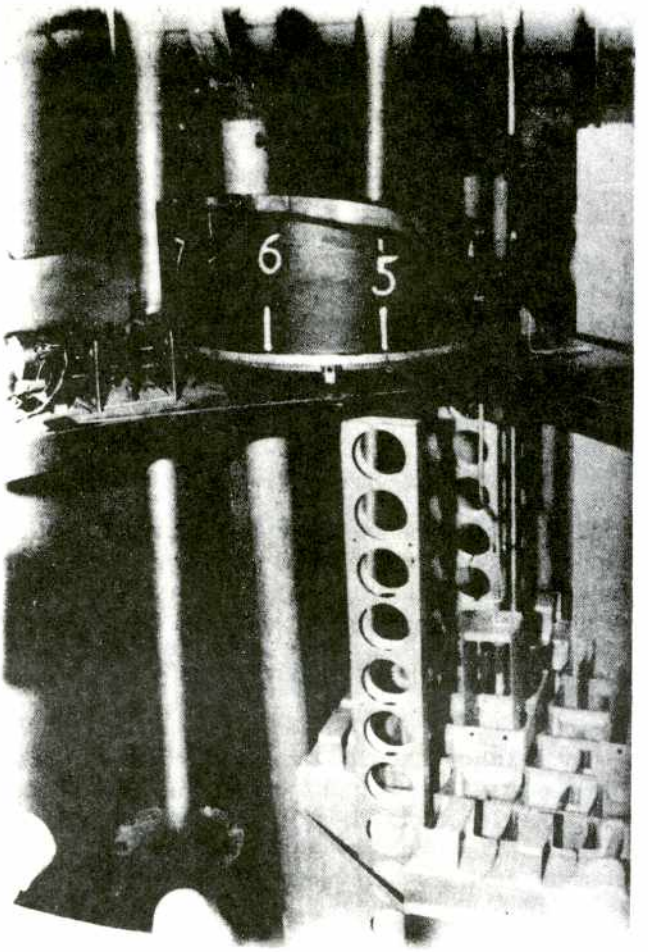


Fig. 3. Apparatus and function rod inside the core.

The chariot, function rod, and bi-prism weigh ~0.600 kg, and they must follow the function leaning on the crown in such a way that the torque that must be supplied by the motor depends strongly on the shape function. To permit such torque variation with constant speed, a speed control for the motor was included.

The display of the reactivity function was, in a first attempt, performed with an electrical resistance and a slide, but it introduced a loud noise; consequently, it was replaced by the bi-prism, as shown in Fig. 2. The two parts of the bi-prism have the same refraction index to avoid displacement of the light beam. Due to imperfections on the acrylic surfaces, some noise is still present, but it can be reduced by means of a capacitor.

The intensity of light reaching the photo-diode after going through the bi-prism is given by

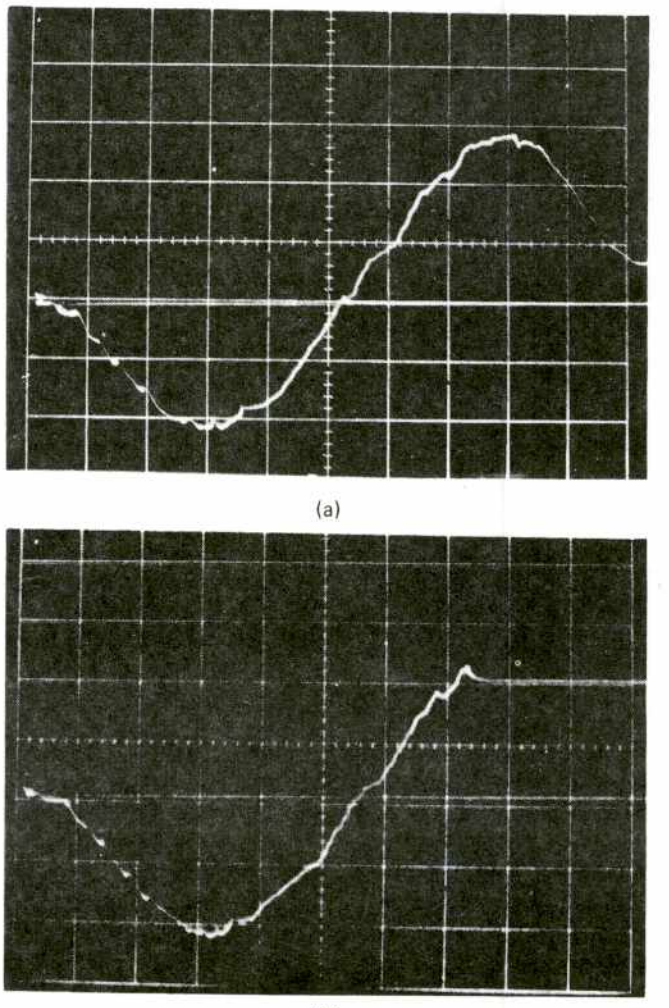
$$I = I_0 \exp [k\rho (t)] \quad , \qquad (3)$$

where I_0 and k depend on monitor properties, $\rho (t)$ being the reactivity function.

Another alternative for monitoring, instead of the bi-prism, could be a strip of film (Super 8), especially when high frequencies are involved in the reactivity function, since it implies a smaller mass to move. This strip of film could be exposed using the same generator, with some special arrangement, so that the image on the film is capable of a linear response instead of an exponential one. However, we did not find the strip of film necessary for the present.

RESULTS AND CONCLUSIONS

A photograph of a monitor display that belongs to a reactivity function whose shape is similar to that shown in Fig. 1 is shown in Fig. 4a. This display was obtained with the crown moving at 0.01 turns per second. The sources of noise present in the curve are of two kinds. One, as



(a)

(b)

Fig. 4. (a) Reactivity function wave form displayed by the monitor system. (b) The same with an accidental stop of the generator.

said before, arises from imperfections in polishing the acrylic surfaces of the bi-prism. We presume that a bi-prism made of glass would eliminate much of this noise, but glass is more difficult to cut and polish. The second source of noise arises from the microphonic effect. The first time a collimated beam of light was used to excite the photo-diode, the collimation was found to be very sensitive to vibrations; hence, it was replaced by the isotropic light emission produced by a point lamp, so that the vibrations of the lamp do not incite the photo-diode, almost to the same extent as the collimated beam.

Figure 4b shows the same curve as before, but with an "accidental" stop at a point of positive reactivity; since the reactivity function is known before operation, the monitor performs an additional task for the safety system. Motor and speed control show a very acceptable performance: The speed variation was found to be on the order of 2% with a load of 3 kg for the crown shaped like the curve of Fig. 1.

Two experimental trials were performed in the RA-2 zero-power reactor; the input reactivities are shown in Figs. 5a and 6a; they were calculated with both inverse kinetics and the BEAT Fortran code[2] to obtain a sinusoidal wave form for the neutron flux, in the former case with an amplitude of 30% above its mean value, and in the second case with an amplitude of 70%. In both cases the frequency was 0.01 Hz.

Choosing this low frequency is based on the desire to analyze the reactor response with regard to time evolution and stability for periodic input reactivity of larger values. For high frequencies, the reactor response can be followed in a short time, because the amplitude peaks follow a line parallel to the mean value.[1]

The function rod consisted of two stainless-steel plates surrounded by cadmium plates, and the amount of this absorber depends on the reactivity amplitudes. The inverse kinetics method was used to calibrate the fine control rod and provided the reference for the function rod as well as the means to find the linear zone of the core.

Turning to the reactor output and assuming that

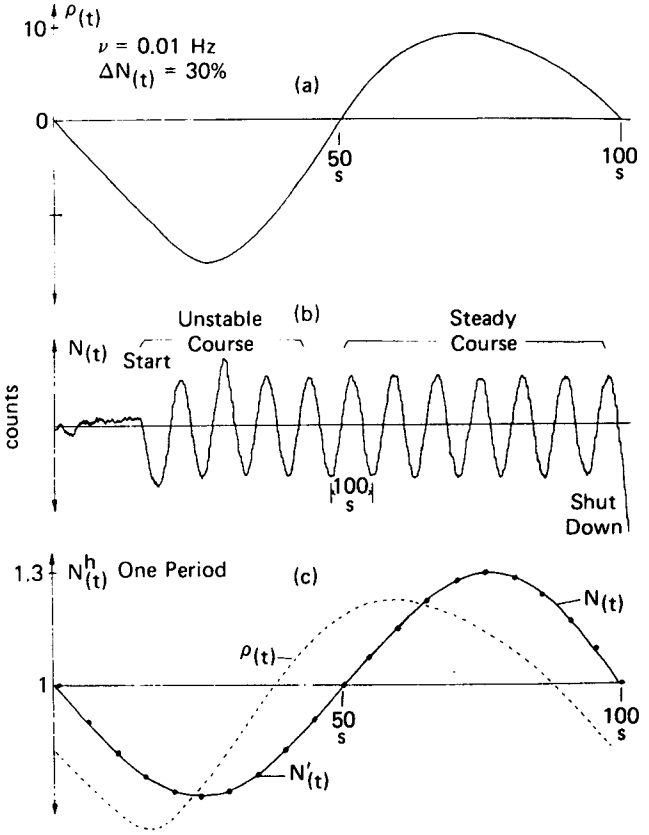


Fig. 5. (a) Reactivity function for a sinusoidal output with an amplitude value of 30%. (b) Time evolution of the reactor output. (c) The solid line, $N_{(t)}$, belongs to the reactor output, with the dots representing its first harmonic (sinus function).

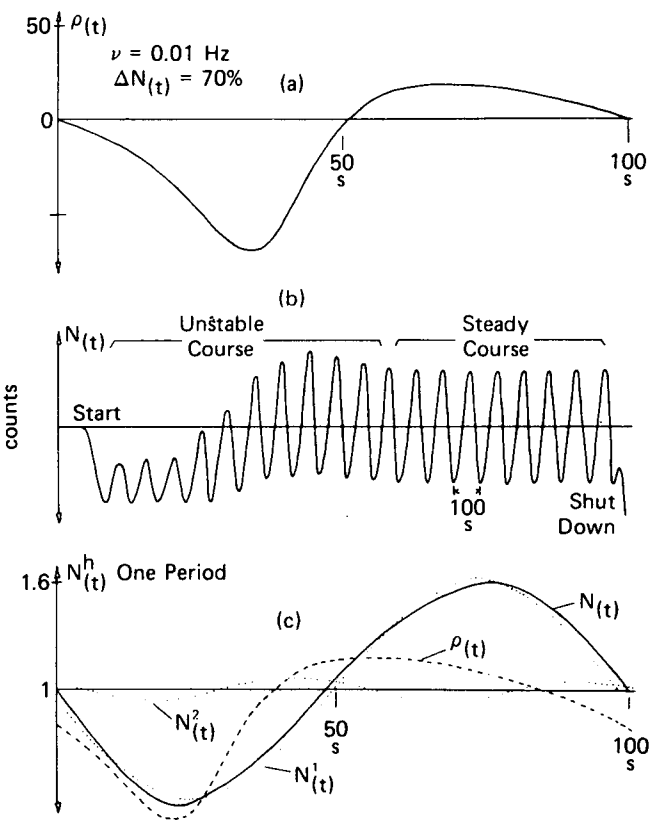


Fig. 6. (a) Reactivity function for a sinusoidal output with an amplitude value of 70%. (b) Time evolution of the reactor output, with the reactivity amplitude values altered. (c) Lines are: $N_{(t)}$, the reactor output; $N^1_{(t)}$, its first harmonic; and $N^2_{(t)}$, its second harmonic.

it is sinusoidal, we can write for the more restricted case,

$$N_{(t)} = \frac{n_{(t)}}{n_0} = 1 + N_0 \sin wt \quad , \tag{4}$$

where n_0 is the neutron density for the critical state and $n_{(t)}$ the density for all the time during evolution. Solving the kinetics equations, we can find the expression of the reactivity function[3] that makes the reactor oscillate according to Eq. (4). We also assume normalization of the delayed-emitter concentration, i.e.,

$$C_i = \frac{c_i}{c_{i0}} \quad ; \quad c_{i0} = \frac{\beta_i N_0}{\lambda_i l} \quad .$$

We start from the kinetics equation given by

$$\rho = \beta + \frac{1}{N} \left(l\dot{N} - \Sigma \beta_i C_i \right) \quad , \tag{5}$$

where C_i is given by the expression

$$C_i = \exp(-\lambda_i t) \left[1 + \int_0^t \exp(\lambda_i \xi) N_{(\xi)} \, d\xi \right] \quad . \tag{6}$$

Inserting Eq. (4) in Eq. (6) and solving, the result for $t > 0$ is

$$C_i = 1 + \frac{\lambda_i N_0}{w^2 + \lambda_i^2} \left[\lambda_i \sin wt - w \cos wt + w \exp(-\lambda_i t) \right] \quad . \tag{7}$$

Substituting in Eq. (5), we obtain

$$\rho = \frac{N_0}{N} \left[\Sigma \frac{\beta_i w^2}{w^2 + \lambda_i^2} \sin wt + \left(l + \Sigma \frac{\beta_i \lambda_i}{w^2 + \lambda_i^2} \right) w \cos wt - w \Sigma \frac{\beta_i \lambda_i}{w^2 + \lambda_i^2} \exp(-\lambda_i t) \right] \quad , \tag{8}$$

this being the reactivity function for a sinusoidal output given by Eq. (4) that must be imposed for $t > 0$ to a reactor critical for $t < 0$. A more general form can be derived for the last expression, since, if we want to start the sinusoidal output with an arbitrary phase, we rewrite Eq. (4) in the form

$$N_{(t)} = a + b \sin(wt + \phi) \quad , \tag{9}$$

where b is again the amplitude oscillation above the mean value a. The values for a and b will depend on the values for the phase ϕ and the ratio b/a. If we write $b/a = N_0$, where $0 < N_0 < 1$, for $t = 0$ we have

$$a + b \sin\phi = 1 \quad , \quad b = N_0 a \quad . \tag{10}$$

Then Eq. (9) results:

$$N_{(t)} = \frac{1}{1 + N_0 \sin\phi} + \frac{N_0}{1 + N_0 \sin\phi} \sin(wt + \phi) \quad , \tag{11}$$

which says that the reactor will oscillate sinusoidally about the mean given by $1/(1 + N_0 \sin\phi)$ with amplitude $N_0/(1 + N_0 \sin\phi)$; at $t = 0$, the reactor starts with $N = 1$ and phase ϕ.

Equations (7) and (9) now become

$$C_i = a + (1 - a) \exp(-\lambda_i t) + \frac{b\lambda_i}{w^2 + \lambda_i^2} \times \left[\lambda_i \sin(wt + \phi) - w \cos(wt + \phi) + (w \cos\phi - \lambda_i \sin\phi) \exp(-\lambda_i t) \right] \tag{12}$$

$$\rho = \frac{N_0}{1 + N_0 \sin(wt + \phi)} \times \left[\Sigma \frac{\beta_i w^2}{w^2 + \lambda_i^2} \sin(wt + \phi) + \left(l + \Sigma \frac{\beta_i \lambda_i}{w^2 + \lambda_i^2} \right) w \cos(wt + \phi) - \sin\phi \, \Sigma \frac{\beta_i w^2}{w^2 + \lambda_i^2} \exp(-\lambda_i t) - w \cos\phi \, \Sigma \frac{\beta_i \lambda_i}{w^2 + \lambda_i^2} \exp(-\lambda_i t) \right] \quad , \tag{13}$$

which for $\phi = 0$ reduces to Eq. (8). The amplitude factor can also be written as

$$\frac{N_0}{1 + N_0 \sin(wt + \phi)} = \frac{aN_0}{a + aN_0 \sin(wt + \phi)} = \frac{b}{N} \quad .$$

Difficulty arises for $t = 0$, in accordance with the exponential terms in both Eqs. (8) and (13), since, if we want a stable evolution of the neutron flux after $t = 0$, a nonperiodic reactivity function of almost exponential shape must be used during the start time interval, other than the periodical one described by the crown. The amount of this exponential reactivity can be strongly reduced if we choose a phase angle such that it is zero at the start time. Taking into account our reactor parameters, this phase angle is given by

$$tg\phi = - \frac{\Sigma \frac{\beta_i \lambda_i}{w^2 + \lambda_i^2}}{w \Sigma \frac{\beta_i}{w^2 + \lambda_i^2}} \quad \therefore \quad \phi \cong \begin{cases} +133 \text{ deg} \\ -47 \text{ deg} \end{cases} \quad . \tag{14}$$

In this case, we have to start with an initial amount of reactivity given only by $(b/N) lw \cos\phi$, which is small for low frequencies, and at the same time, we have to describe the exponential reactivity ("exponential" as a name) whose amplitude is not very different from zero at all times,[a] whenever the phase angle is given by Eq. (14).

Conversely, if we start the oscillation with a

[a] The exponential reactivity was computed over ten periods, the results showing that its minimum root-mean-square value belongs to the phase angle given by Eq. (14).

reactivity value given by $(b/N)lw\cos\phi$, even neglecting the exponential reactivity, the reactor response will have a phase angle similar to Eq. (14). This initial reactivity value can be checked as follows: Applying general considerations to an arbitrary neutron flux, for that small reactivity and a short interval of time after critical, we have

$$N_{(t)} = N_{t=0} \times \exp \frac{1}{l}\left(\frac{b}{N} l w \cos\phi\right)\Delta t \quad .$$

If, as is the case, $N_{t=0} = 1$, then

$$\Delta N = \exp(bw\cos\phi)\Delta t - 1 \quad ,$$

which implies that the initial slope is

$$\dot{N}_{t=0} = bw\cos\phi \quad ,$$

this being the same slope as the function given by Eq. (9).

As for Eq. (12), the delayed emitter concentration when t is large will be given by

$$C_i = a + b\left[\frac{\lambda_i^2}{w^2+\lambda_i^2}\sin(wt+\phi) - \frac{\lambda_i w}{w^2+\lambda_i^2}\cos(wt+\phi)\right] \quad .$$

Substituting a from Eq. (9), we get

$$\Sigma\beta_i C_i = N\beta - b\left[\Sigma\frac{\beta_i w^2}{w^2+\lambda_i^2}\sin(wt+\phi) + \Sigma\frac{\beta_i\lambda_i w}{w^2+\lambda_i^2}\cos(wt+\phi)\right] \quad .$$

Taking into account Eq. (14) for times such that $wt = 2\pi n$, the latter reduces to $\Sigma\beta_i C_i = N\beta$ and the reactivity is $(b/N)lw\cos\phi$; introducing these results in the general equation given by

$$l\dot{N} = \rho N + (\Sigma\beta_i C_i - \beta N) \quad ,$$

we find that

$$\dot{N} = bw\cos\phi \quad ,$$

which shows that in times when the reactivity takes values close to zero, the neutron flux behaves as if all neutrons were prompt, or, alternatively, if we return to the not normalized kinetics equations, the expression $\Sigma\beta_i C_i - \beta N = 0$ implies that $\Sigma n(\beta_i/l)\Sigma\lambda_i C_i$, i.e., at the "intersection times," the total rate of formation of all the delayed neutrons equals the total rate of formation of all the precursors. For instance, one of the two intersection times is set where the decreasing reactivity intersects the value labeled "1" in Fig. 5c; at this time, the sinusoidal output is also decreasing, and the sign of $\Sigma n(\beta_i/l) - \lambda_i C_i$ inverts from positive to negative. In the other intersection time, the situation is reversed. Therefore, we can say that the start time discussed above is equivalent "to hook" the stable response (for large t) in those times, with the reactivity $(b/N)lw\cos\phi$.

Turning to the experimental part, once the reactor was in the critical state, we started with decreasing reactivities (from point 0 to the right in Fig. 5a); then, with the fine control rod (unstable course in Fig. 5b), the mean value of the response was conducted to that of the critical state (steady course). It is from this last course that we have taken the information to analyze. The curve of Fig. 5b is actually that obtained from a linear graphic recorder.

The output signal from the reactor was fed through an amplifier to an FM tape recorder, where all the inherent noise was present. The resultant curve was found to be composed of the sinusoidal response plus a relative strong neutronic noise.

As our attention was directed toward the harmonic of 0.01 Hz, the recorded signal was filtered through the analog computer operating with four RC steps filters, providing the curve with harmonics whose amplitudes are given by

$$|A_i| = (1 + 4\pi^2\nu_i^2)^{-n/2} \quad , \qquad (15)$$

where n is the steps number (in our case $n = 4$) and ν_i is the frequeney of the i'th harmonic.

The curve so obtained was then processed with the BEAT code[2] to separate it into its harmonics and phase angles, which were normalized with Eq. (15).

The result for the first experiment (flux variation = 30%) is shown in Fig. 5c: the solid line labeled $N_{(t)}$ is the one full wave of the neutron flux after it is filtered; the dots labeled $N^1_{(t)}$ represent its first harmonic component. (Harmonics other than the first are insignificant.) Analysis of both curves shows that the response is sinusoidal to a very good extent since, if we define distortion as

$$D = \frac{1}{A_1}\left(\sum_{i=2}^{\infty} A_i^2\right)^{1/2} \quad ,$$

where A_i are the coefficients of the expression

$$N = 1 + \Sigma A_i\cos(w_i t - \alpha_i) \quad , \qquad (16)$$

the distortion for the curve of Fig. 5c results in $D = 3\%$ (see Table I), which includes all of the experimental errors.

The dotted line in Fig. 5c again represents the reactivity function, but with the proper phase relative to the neutron flux. Figure 6 describes the second experiment, in which the reactivity function should produce a sinusoidal output with $N_0 = 0.7$ in Eq. (4); although the reactivity function had the correct shape, the reactivity values were

TABLE I

Constants for Eq. (16)

$\Delta N = 30\%$		$\Delta N = 70\%$	
A_i	α_i	A_i	α_i
0.298	-1.4°	0.588	-10°
0.007	50	0.054	112
0.005	26	0.008	87
0.003	26	0.012	30
$D = 3\%$		$D = 10\%$	

80% of those calculated, i.e., instead of 0.19 dollars of positive amplitude, we obtained 0.15, which implies some degree of distortion in the response to be expected.

Figure 6a shows the reactivity function with the proper shape and values. As before, Fig. 6b shows the whole response in time.

The initial depletion of the curve at the start obeys conservation because, since the reactivity is now bigger, we started with the reactor critical but with reactivity "before the zero." Then, as in the previous experiments, the neutron level was induced to oscillate about the initial critical state.

Figure 6c shows one period of the response represented by the solid line, with the first and second harmonics represented by the dotted lines.

The degree of distortion is evident here, this being $D = 10\%$ according to Table I, which shows that the shape output is very sensitive to that of the input. The output amplitude has the value of 60%, instead of 70%, due to the lower value of the reactivity input, although the shape of the latter was the correct one.

All calculations were based on the point model reactor, and the results discussed above show a very good agreement in these experiments. Frequencies other than 0.01 Hz were used in a previous study,[3] with similar performance, although they were conducted with different purposes.

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REFERENCES

1. J. PORTO, "Analysis of Dynamic Perturbation in Zero Power Reactor," *Ann. Nucl. Sci. Eng.*, **1** (1974).

2. C. GRANT and J. PORTO, "BEAT: Fortran Code for Neutron Kinetics," *News from CPL* (Apr. 1976).

3. RAYMOND L. MURRAY, CARROLL R. BINGHAM, and CHRESTON F. MARTIN, "Reactor Kinetics Analysis by an Inverse Method," *Nucl. Sci. Eng.*, **18**, 481 (1964).