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Graphical multistep shell-model calculation of the even tin and lead ground states

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The ground-state energies and wave functions of tin and lead even isotopes are calculated within a multistep shell-model method and good agreement with experimental data is obtained. To carry out this calculation a general graphical procedure is introduced.

[NUCLEAR STRUCTURE Calculated ground states of tin and lead isotopes.]

It was recently shown that the low-lying levels of nuclei with six particles outside closed-shell cores can be described in terms of the low-lying levels of the corresponding two- and four-particle nuclei, thus allowing drastic truncation of the corresponding shell-model basis.[1] One proceeds in several steps. First, the two-particle system (which defines the two-particle interaction) is solved. Second, the four-particle system is solved in the basis formed by vector coupling the two-particle states previously evaluated. Finally, the six-particle system is calculated within a basis which consists of the two- and four-particle vector-coupled states. Within this multistep shell-model method (MSM) both the six-particle dynamical and metric equations turn out to be rather simple, essentially because they only depend upon two- and four-particle quantities.

In this paper we present a generalization of the MSM. We thus consider a system σ consisting of a number s of particles. We assume σ partitioned in the subsystems μ and ν with numbers of particles m and n, respectively, such that $s=m+n$. A general state $|g\rangle$ is written as $|g\rangle=P^+(g)|0\rangle$, where $|0\rangle$ is the core wave function. For the state $|\sigma\rangle$ one has

$$P^+(\sigma)=(1+\delta_{\mu\nu})^{-1}\sum_{\mu\nu}Y(\mu\nu;\sigma)P^+(\mu)P^+(\nu)\,, \qquad (1)$$

where $Y(\mu\nu;\sigma)=(1+\delta_{\mu\nu})X(\mu\nu;\sigma)$. The wave function amplitude X is not a well-defined quantity, when one deals with overcomplete bases.[1] However, in the cases treated below X is well defined. The dynamical equation corresponding to the partition (1) is obtained from

$$\Lambda(\mu\nu;\sigma)=\left\langle\sigma\left|\left[[H,P^+(\mu)],P^+(\nu)\right]\right|0\right\rangle$$
$$=(W_\sigma-W_\mu-W_\nu)\langle\sigma|P^+(\mu)P^+(\nu)|0\rangle\,, \qquad (2)$$

where W is energy referred to the core. The double commutator in Eq. (2) can be calculated by rewriting the operators P^+ in terms of their components [as in Eq. (1)] until one reaches the single-particle creation operators. This calculation can be very tedious. Moreover, the two-particle interaction appears generally in combinations that allow (2) to be written in terms of the energies and wave functions of the μ and ν systems. But when the

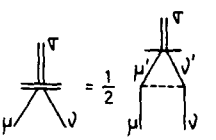

FIG. 1. The double bar vertex represents the two-particle coupling constant, Eq. (3). The one bar vertex represents the overlap $\langle\sigma|c^\dagger_\mu c^\dagger|0\rangle$ (in general, $\langle\sigma|P^+(\mu')P^+(\nu')|0\rangle$) and the dashed line the two-particle interaction. In this and all other figures the single-particle states are represented by one line while other states are represented by double lines.

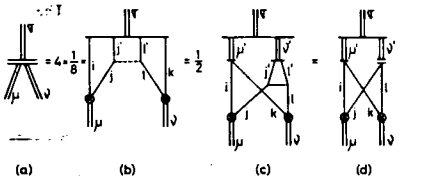

FIG. 2. Graphical calculation of the coupling-constant Eq. (2) for the four-particle state σ in terms of the two-particle states μ and ν.

basis is overcomplete, there might be many different possibilities to write such an equation, and it may not be easy to find the most convenient of those possibilities. To overcome these difficulties one can utilize a graphical method to evaluate (2). Thus, for $n=m=1$ one has

$$\Lambda(\mu\nu;\sigma)=\tfrac{1}{2}\sum_{\mu'\nu'}\langle\mu'\nu'\,|\,V\,|\,\mu\nu\rangle\langle\sigma\,|\,c_{\mu}^{+}c_{\nu}^{+}\,|\,0\rangle\ , \tag{3}$$

where we have used c^+ (instead of P^+) for the fermion creation operators, as usual. The quantity Λ in Eq. (3) is the two-particle (TDA-boson) coupling constant of the nuclear field theory.[2] We use the name "coupling constant" even for Λ in the general case of Eq. (2). The graphical representation

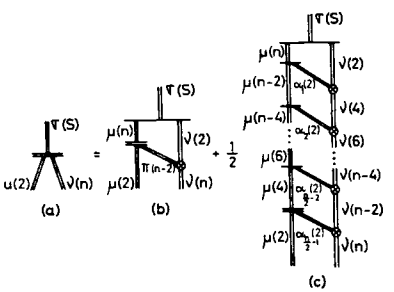

FIG. 3. Graphical representation of the coupling constant (2) for the partition $s=2+n$, $n>4$, and even. Many other forms are possible for this coupling constant. We assumed here that all intermediate states (i.e., steps of the MSM) are also partitioned as $n=2+(n-2)$, as seen in the figure. The number of particles associated with each state is explicitly given (in parenthesis) in the labeling. As an illustration, the analytical expression corresponding to graph (b) is (summation goes over repeated indices) $Y[\pi(n-2)\nu(2);\nu(n)]\times\Lambda[\mu(2)\pi(n-2);\mu(n)]\,\langle\sigma(s)\,|\,P^{+}[\mu(n)]P^{+}[\nu(2)]\,|\,0\rangle$.

of Eq. (3) is given in Fig. 1. In Fig. 2 is given the graphical representation of Λ for $s=4$ and $m=n=2$. In Fig. 2(b) the two-particle states μ and ν are decomposed into their single-particle components [the crossed circles represent the amplitudes Y of Eq. (1)]. There is a factor 4 because there are four ways of joining two lines through the interaction giving the same contribution to Λ. The factor $\frac{1}{8}$ is given by the interaction $\frac{1}{2}$ and the δ function in Eq. (1) $\frac{1}{4}$. In Fig. 2(c) projectors $\sum_{\mu}|\mu\rangle\langle\mu|=1$ have been introduced between pairs of single-particle lines to allow us, using Fig. 1, to write the four-particle coupling constant in terms of two-particle energies and wave functions. The resulting analytical expression is, from graphs 2(a) and 2(d), $\Lambda(\mu\nu,\sigma)=(W_{\sigma}-W_{\mu}-W_{\nu})\langle\sigma\,|\,P^{+}(\mu)P^{+}(\nu)\,|\,O\rangle=-\sum_{\mu'\nu'}\sum_{ijkl}X(ij,\mu)\times X(kl,\nu)\langle\mu'\,|\,c_i^{+}c_k^{+}\,|\,0\rangle\Lambda(jl,\nu')\,\langle\sigma\,|\,P^{+}(\mu')P^{+}(\nu')\,|\,0\rangle$. The minus sign is because two single-particle lines are crossed, that is, two fermion operators have been exchanged. The coupling to good angular momenta can be made using any of the available graphical methods (e.g., Ref. 3). In any other case one proceeds as in Fig. 2. The graph itself makes it apparent which is the best possible choice to evaluate Λ. Thus, in Fig. 2 it is evident which lines have to be joined through a "bar-vertex" in order to replace the two-particle interaction by the coupling constant given in Fig. 1.

We applied this graphical procedure to calculate the dynamical matrix of a system with a number $s=2+n$ of particles where n is any even number. For $s>6$ one obtains the graphs shown in Fig. 3. The cases $s\leq 6$ were already treated in Ref. 1, where it was found that the ground-state wave functions in the lead region could be written as

$$|\,\sigma(\text{g.s.};s)\rangle=N_{\sigma}P^{+}(\text{g.s.};2)P^{+}(\text{g.s.};n)\,|\,0\rangle\ , \tag{4}$$

where the normalization constant N_{σ} is the amplitude X of Eq. (1). Assuming that all intermediate states $\mu(n)$ and $\nu(n)$ in Fig. 3 are also well described by one-component wave functions of the form of Eq. (4), one can then easily evaluate the ground-state energy W_{σ}. One obtains

$$W_{\sigma}=W_{\sigma}(a)+W_{\sigma}(b)+W_{\sigma}(c)\ , \tag{5}$$

where $W_{\sigma}(a)=W_{\text{g.s.}}(2)+W_{\text{g.s.}}(n)$; $W_{\sigma}(b)=W_{\text{g.s.}}(n)-W_{\text{g.s.}}(2)-W_{\text{g.s.}}(n-2)$; $W_{\sigma}(c)=W_{\text{g.s.}}(4)-2W_{\text{g.s.}}(2)$, and $W_{\text{g.s.}}(m)$ is the ground-state energy of the m-particle system. The three contributions in Eq. (5) correspond to the three graphs in Fig. 3 [actually the contribution of graph 3(a) is $W_{\sigma}-W_{\sigma}(a)$]. The factor $\frac{1}{2}$ in Fig.

TABLE I. Theoretical and experimental (Ref. 4) ground-state energies (in MeV) in the tin and lead regions. The number of active particles is n. For $n > 6$ we applied Eq. (5) while $n = 6$ was taken from the one-element basis of Ref. 1. The theoretical zeroth order energy $W_\sigma(a)$ is given together with the correction $W_\sigma(b)$. The contribution $W_\sigma(c)$ is 0.930 MeV in Sn and 0.714 MeV in Pb. Some "experimental" data are actually estimated from systematic trends. In this case, rather large errors (up to 1 MeV) may be encountered (Ref. 4). In addition, the binding energy of the core ^{132}Sn is only known within an error of 200 keV.

	^{A}Sn ($A = 132-n$)				^{A}Pb ($A = 208-n$)			
n	$W_\sigma(a)$	$W_\sigma(b)$	W_σ (theor)	W_σ (expt)	$W_\sigma(a)$	$W_\sigma(b)$	W_σ (theor)	W_σ (expt)
6	37.380		39.240	38.796	43.035		44.463	44.246
8	50.946	1.416	53.292	52.722	58.353	1.211	60.278	60.170
10	64.872	1.776	67.578	67.159	74.277	1.817	76.808	76.570
12	79.309	2.287	82.526	82.146	90.677	2.293	93.684	93.470
14	94.296	2.837	98.063	97.737	107.577	2.793	111.084	110.950
16	109.887	3.441	114.258	114.008	125.057	3.373	129.144	128.610
18	126.158	4.121	131.209	131.117	142.717	3.553	146.984	146.820
20	143.267	4.959	149.156	149.162	160.927	4.103	165.744	165.690
22	161.312	5.895	168.137	168.129	179.797	4.763	185.274	185.000
24	180.279	6.817	188.026	188.200	199.107	5.203	205.024	
26	200.350	7.921	209.201	209.260				
28	221.410	8.910	231.250	231.240				
30	243.390	9.830	254.150					

3(c) is canceled by the factor 2 in the crossed vertex that opens the line $\nu(4)$ (i.e., $Y[\text{g.s.}(2)\text{g.s.}(2); \text{g.s.}(4)] = 2N_{\sigma=\text{g.s.}(4)}$). Notice that Eq. (5) does not depend upon the normalization constants. They are canceled by the corresponding bar vertices in Fig. 3 [i.e., from Eqs. (1) and (4): $N_\sigma\langle\sigma|P^+(\text{g.s.};,2)P^+(\text{g.s.};n)|0\rangle = 1$].

In Table I we give both the experimental and calculated ground-state energies in the lead and tin regions. Taking into account the great differences between the zero order energies $W_\sigma(a)$ and the experimental values, one can well say that the agreement between theory and experiment is good. In the middle of the major shells, where the shell-model dimension is of the order of several millions, this agreement is excellent.

Finally, it is worthwhile to point out that the matrix can also be evaluated with the graphical method outlined here.

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