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Dans ce numéro

ON THE CONCEPT OF TIME, BIOLOGICAL TIME, AND MODELLING SUBJECTIVE TIME (by M.Mashour, Sweden)

ENVIRONMENTAL STABILITY
by C.Léguizamon (Argentine)

PLACE DES ETRES ORGANISES SUR L'ECHELLE DE TEMPS par C.Bogdanski (Paris)

RYTHME CIRCADIEN, INFORMATION ET TEMPS BIOLOGIQUE CHEZ LA PLANTE par P.Franquin (Paris)

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ENVIRONMENTAL STABILITY

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RESUME

Le système de l'environnement consideré comme un système qui ne contient pas d'autres systèmes biologiques, a été représenté par l'auteur (1975 b) à travers développements relationnels qui ont eté formalisé par la suite, avec la théorie des catégories.

Chaque système de l'environnement E_i, est obtenu à l'aide de'un postulé générale qui établit que pour une période de temps quelconque d'un système biologique M_i donné, el existe E_i qui est le seul système directement en rapport avec lui.

De cette façon, la suite biologique (1) donne lieu à la suite de l'environnement (2) et toutes les deux se correpondent, pour des lapses infinitesimales avec l'axe qui exprime la variable temps. Ceci donne lieu a une interpretation dynamique, dans le futur, de la stabilité qui est définie.

Chaque matière mise en jeu, avec son énergie extrinsèque associée, est représentée par des objets de la catégorie E_i qui formalisent le système de l'environnement respectif. Les morphismes identifient des comportements chimiques, mécaniques, théoriques,

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etc., parmi ses domaine et codomaine respectifs.

Les différents aspects de stabilité que l'on définit sont compris sur la base des structures qui sont conservées constantes à travers les catégories de l'environnement de la suite (2). Ces structures sont sous-catégories qui sont définies par le domaine et codomaine de fonctors. On exige que les fonctors soient isomorphiques, ce que est une conséquence du criterium de constance déjà mentionné.

La définition D.1 caractérise deux sous-catégories E_i^d (de E_i) et E_{i+1}^c (de E_{i+1}) qui sont isomorphiques par le fonctor défini dans (3). En plus, on exige que E_i^d et E_{i+1}^c soient déterminées par le maximum nombre d'objets communes a E_i dans E_{i+1}, bien que, pour certains cas, le criterium d'isomorphicité impose des morphisms vides entre des paires d'objets déterminés (condition D.1b)

Le fonctor (3) répété entre des catégories de l'environnement successives permet de définir à travers D.2 la sous-catégorie nucléus E_i^o qui est définie comme sous-catégorie de E_i^d et E_i^c. La condition D.2a exige aussi une cardinalité maximum dans la collection d'objets de E_i^o, bien que pour ceci peuvent être necessaires des morphismes vides entre déterminées paires d'objets (condition D.2b).

On définit aussi une sous-catégorie contribution $\mathcal{L}_i$ (définition D.5) qui est basiquement définie par le fonctor isomorphique (4), dont le codomaine $\mathcal{L}_{i+1}^{(i)}$ est exigé comme sous-catégorie pleine de la sous-catégorie nucléus E_{i+1}^o. En plus, $\mathcal{L}_i$ est sous-catégorie de E_i^i qui a son tour, pour D.3, est sous-catégorie pleine de E_i^d et qui est déterminée par des objets de E_i^d qui n'appartiennent pas a E_i^o.

A cause de ceci, E_i^i considère tous le objets qui entrent dans la respective catégorie de l'environnement E_i, de telle façon que $\mathcal{L}_i$ considère que les objets qui entrant dans E_i font aussi part de la catégorie nucléus E_{i+1}^o du système de l'environnement successive E_{i+1}.

Sur la base de ces définitions, les nouveaux concepts sont exposés. De cette façon, le fonctor isomorphique (5) met en rapport la sous-catégorie nucléus E_{i+j-1}^o d'un système E_{i+j-1} avec la sous-catégorie isomorphique $E_{i+j}^{o(i+j-1)^{i+j-1}}$ qui est une sous-catégorie de la sous-catégorie nucléus E_{i+j}^o dans le système successif. Ainsi, la composition (6) de n fonctors isomorphiques (5) établit que <u>la stabilité S_i du systeme de l'environnement E_i est n</u>, quand la sous-catégorie nucléus E_i^o est le premier domaine de la composition. Dans d'autres mots, une structure E_i^o qui est déjà, au moins, entrée dans un système E_{i-1} antérieur comme E_{i-1}^d par le fonctor (3), est maintenue définie comme telle par sous-catégories E^d et E^c, à travers n systèmes postérieurs à E_i.

A travers n compositions du type (7) du fonctor (4) l'on obtient que, puisque $\mathcal{L}_i$ est le domaine initial de la composition et que $\mathcal{L}_i$ représente des objets qui entrent dans E_i qui conservent après la structure dans des sous-catégories nucléus successives E_{i+j}^o ($j= 1,2,\dots, n$), alors on dit que la <u>contribution $\mathcal{A}_i$ du système E_i à la stabilité de la chaîne de l'environnement est n</u>.

Puisque l'on peut définir p sous-catégories $\mathcal{L}_{i+s}^{(\alpha)}$ différentes dans E_{i+s}^o à travers p compositions du type (7), alors on définit que le <u>degré de contri-</u>

bution $\mathcal{G}_{i+s}$ dans le système de l'environnement E_{i+s} est p.

Une série de theorèmes postérieurs assez simples établissent la dynamique des objets qui entrent et sortent à travers la chaîne de l'environnement, par rapport à l'appartenance des sous-catégories définies pour établir les concepts de stabilité. Les résultats établissent basiquement: a) Les objets qui appartiennent à sous-catégories nucléus sont les seuls qui peuvent apparaître dans la même sous-catégorie d'un système suivant, et b) pas tous les objets qui entrent dans la chaîne de l'environnement aident à la stabilité.

Le travail aussi présente une introduction dans laquelle on fait mention de l'utilité de ces concepts pour analyser des comportements biologiques sur la base de la stabilité de l'environnement que l'on définit.

Dans la discussion postérieure, le travail fait ressortir l'existence d'aspects temporels entre les trois nouveaux concepts de stabilité qui ont été développés.

SUMMARY

The environmental system considered as non-including other biological systems, was represented by the author (1975 b) through relational criteria, later formalized with the theory of categories.

The obtainment of each environmental system E_i, is due to a general postulate which establishes that for any period of time of a given biological system M_i, there exists E_i as its unique directly related system.

By this way, the biological succession (1) gives place to the environmental succession (2) and both correspond for infinitesimal lapses with the axis that expresses the variable time, in such a manner that a dynamic interpretation may be possible in the future.

Each matter involved and its associated extrinsical energy are represented by objects of the E_i category which formalizes the respective environmental system. The morphisms identify chemical, electric, mechanical, theoretical and other behaviors between their respective domain and codomain.

The criteria of stability that are defined, are understood on the basis of the structures that remain constant through the environmental categories of the succession (2). These structures are subcategories which are defined by the domain and codomain of functors. These functors are required as isomorphic, which comes from the criterion of constancy already mentioned.

The definition D.1 characterizes two subcategories E_i^d (of E_i) and E_{i+1}^c (of E_{i+1}) which are isomorphic for the functor defined in (3). Besides, E_i^d and E_{i+1}^c are required to be determined for the maximum number of common objects of E_i in E_{i+1}, in spite of the fact that, for certain cases, the isomorphic criterion for functors imposes empty morphisms between determined pairs of objects (condition D.1b).

The functor (3) when repeated among successive environmental categories permits to define by means of D.2, the subcategory nucleus E_i^o, which is defined as subcategory of E_i^d and E_i^c. The condition D.2a also requires maximum cardinality in the collection of objects of E_i^o, although, for this, empty morphisms between determined pairs of objects may be necessary (condition D.2b).

A subcategory contribution $\mathcal{L}_i$ (definition D.5) is also defined, which is basically defined by the isomorphic functor (4), for which codomain $\mathcal{L}_{i+1}^{(i)}$ it is required as full subcategory of the subcategory nucleus E_{i+1}^o. Besides, $\mathcal{L}_i$ is subcategory of E_i^i. By means of D.3, E_i^i is full subcategory of E_i^d and is determined by objects of E_i^d that do not belong to E_i^o. Because of this later , E_i^i takes into account all the objects going into the respective environmental category E_i. Moreover if some of these objects also belong to $\mathcal{L}_i$, then they will become part of the category nucleus E_{i+1}^o of the successive environmental system E_{i+1}.

So, the criteria of stability are exposed by means of these previous definitions. The isomorphic functor (5) relates the subcategory nucleus

E^{o}_{i+j-1} of a system E_{i+j-1}, with the isomorphic subcategory $E^{o(i+j-1)}_{i+j}$ that is subcategory of the subcategory nucleus E^{o}_{i+j} in the successive system. So, the composition (6) of n isomorphic functors (5) establishes that the stability S_i of the environmental system E_i is n, when the subcategory nucleus E^{o}_{i} is the first domain of the composition.

Under n compositions of the functor (4) in the manner (7), it is obtained that the contribution $\mathcal{A}_i$ of the system E_i to the stability of the environmental chain is n.

As p different subcategories $\mathcal{L}^{(\alpha)}_{i+s}$ may be defined in E^{o}_{i+s} by p compositions of the type (7), then it is defined that the degree of contribution $\mathcal{G}_{i+s}$ in the environmental system E_{i+s} is p.

A serie of simple theorems establishes the dynamic of the objects through the environmental chain. The results basically establish that: a) The objects which belong to subcategories nucleus are the unique ones that may appear in the same subcategory of a subsequent system. b) Not all the objects going into the environmental chain, contribute to the stability.

The paper also presents an introduction about the usefulness of these criteria for analyzing biological behaviors on the basis of the environmental stability that is defined.

In the discussion, the paper emphasizes the existence of temporal aspects among the three fundamental developed criteria.

1.- INTRODUCTION:

It is a known fact that biological and environmental systems are closely related, and that the full knowledge of the environment implies knowledge of the life associated with it and vice-versa.

On this basis, it was ~~as~~ necessary to introduce an environmental theory from the more general viewpoint of Relational Biology (Leguizamón, 1975 b) as it was to develop it with reference to the considerations mentioned above. In addition to this theory, other aspects concerned with the representation of biological systems were developed elsewhere (Leguizamón, 1975 a, 1976), and both environmental and biological formalizations gave rise to a theory of transfers between these systems (Leguizamón, 1977 a). Using a simple mathematical procedure, it was obtained a new bio-environmental system K which comprised all these systems. (Leguizamón, 1977 b). This system enables us to represent mathematically, the existing interdependence of systems. The analysis of this system K or of the biological and environmental systems separately, is important to understand the part played by each one of these interdependent systems in different situations, for instance, that of a living system which has remained unchanged for a long time, or those given by others whose continuous structural changes result in a high diversification of species. According to these generalities, the environment might be interpreted, in the first case, as more constant than that in the second one, and consequently, it would be useful to quantify that constancy in each case.

Making reference to the definition of an "environmentally static biological system" (Leguizamón, 1975 b), which states that such a system has remained in the same environment throughout its life, it immediately suggests a need to know about the constancy of the environmental structure. This structure seems to be permanently changing when a system M is defined as environmentally static. The last assumption could be used for the study of the spontaneity of the origin of life (Leguizamón, 1977 b) which is related to the length of time required for the organization of the environment prior to the apparition of life. The constancy of certain structural parts in the environment before and after life was originated, may also be assumed as having been determinative of the moment when this event took place.

The last considerations point to the importance of infering biological consequences from the analysis of the constancy or variability of the environment.

It can be readily understood that the present work is not only concerned with the biological aspects related to the environmental events, but also with the development of a new criterion of stability. This mathematical concept will arise both from the environmental theory expounded and from the theory of categories which was used to represent the former. (Leguizamón, 1975 b).

Therefore, an interaction between biological and mathematical concepts is requires, though sometimes involving repetitions and apparently trivial considerations, as a mean of gaining a better insight of its possible future applications within either field.

The present definition of stability is understood as a definition of constancy of structure through time, or through successive systems starting from the original system for which this stability is defined. Hence, considering the general postulate proposed in the above mentioned paper from which: "For any period of time of a given biological system M, there exists a unique system directly related to M, such that this system is its associated environmental system E"; then, given a chain of biological systems such as:

$$\ldots M_{i-1} \longrightarrow M_i \longrightarrow M_{i+1} \longrightarrow \ldots \qquad (1)$$

which represents the temporal succession of structures of a unique and general system M or the temporal succession of individual systems; it will be associated with the following environmental chain:

$$\ldots E_{i-1} \longrightarrow E_i \longrightarrow E_{i+1} \longrightarrow \ldots \qquad (2)$$

Thus, given the category E_i, which represents the corresponding environmental system, the constancy of a certain part of its structure, will be given by isomorphic functors between subcategories of the respective categories involved.

Constancy of objects and morphisms through two or more categories must be considered not only from the mathematical point of view, but also from that of the criteria adopted to define those objects and morphisms.

Thus, if objects and morphisms in an environmental category are mathematically identical to objects and morphisms of one or more consecutive environmental categories, they must also be identical from the point of view of the physical reality for which objects and morphisms were defined.

Thus, in a general description, each object of each environmental category E_i; that represents an environmental system E_i not including biological systems into it; represents either a simple set of the form Y_i $(1^{Y}i)$, where a given material (theoretical) physical nature is recognized; (which is strictly equivalent to a given value of intrinsic energy) or a one point set of the form E_i^y $(1^{E_i^y})$, where the extrinsical (theoretical) energy linked to Y_i $(1^{Y}i)$ is represented. A simple cartesian product operation between the two kind of objects defines the linkage between the concepts of matter and energy.

Similarly, each morphism of E_i is the formalization of well determined chemical, electrical, mechanical, theoretical, or other actions taking place between the previous sets.

2.- DEFINITION:

The definition of stability will be obtained by means of isomorphic functors between subcategories of successive environmental categories.

Let us recall the definitions of some concepts of category theory;

T.C.1. Given two categories $\mathcal{B}$ and $\mathcal{C}$, a functor $\mathcal{F}$ such that:

$$\mathcal{T} : \mathcal{B} \longrightarrow \mathcal{C}$$

is an isomorphic from $\mathcal{B}$ to $\mathcal{C}$ if it is a bijection on objects and morphisms.

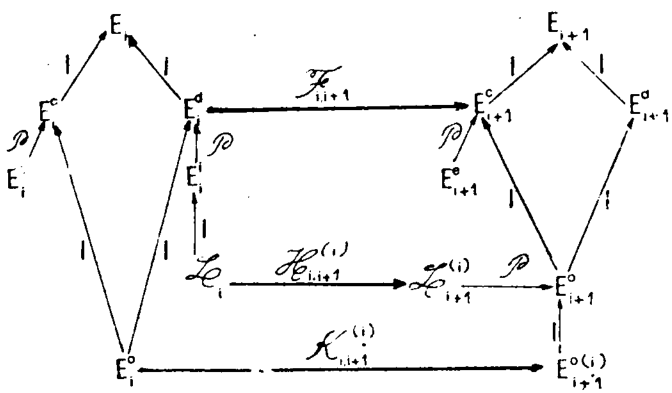
T.C.2. A functor $\mathcal{T} : \mathcal{B} \longrightarrow \mathcal{C}$ is full, when, for every pair of objects B, B' of $\mathcal{B}$, and for every morphism $g : \mathcal{T}(B) \longrightarrow \mathcal{T}(B')$ of $\mathcal{C}$, there is a morphism $f : B \longrightarrow B'$ with $g = \mathcal{T}(f)$

T.C.3. A subcategory $\mathcal{B}'$ of the category $\mathcal{C}$ is full, when the inclusion functor (injection mapping)

$$\mathcal{T} : \mathcal{B}' \longrightarrow \mathcal{C}$$

is full

The diagram shown in Figure 1 will be basic for the comprehension of definitions and of general use in the next developments.



I: inclusion functors

$\mathcal{P}$: full inclusion functors

FIG. 1

Figure 1, which is only apparently complex, illustrates two consecutives environmental systems E_i and E_{i+1}, in which the same subcategories are defined. Thus, an isomorphic functor such as $\tilde{\mathcal{F}}_{i,i+1}$ is the natural, mathematical means of transfer from E_i (as E_i^d) to E_{i+1} (as E_{i+1}^c) of the maximum similarity in structure compatible with the two consecutive systems. With respect to the system E_i, it is almost impossible that E_i would be the same as E_i^d. Then the subcategory E_i^o is defined, such that the collections of objects and morphisms exist both in the preceding system E_{i-1} (by means of E_{i-1}^d) and in the subsequent system E_{i+1} (by means of E_{i+1}^c). Thus, the constancy of structure of E_i^o, at least along three consecutive environmental systems, affords a means of defining stability.

The functor $\mathcal{H}_{i,i+1}^{(i)}$ is defined as another isomorphic functor where the subcategory $\mathcal{L}_{i+1}^{(i)}$ will be a full for the new, basic structures E_{i+1}^o by means of which the stability of the next system E_{i+1} is defined.

As $\mathcal{L}_{i+1}^{(i)}$ has an isomorphic structure $\mathcal{L}_i$ in E_i, and as it represents new environmental units (objects) and behaviors (morphisms) entering system E_i (category E_i^i), then $\mathcal{L}_i$ can be used as a natural, mathematical tool to represent the contribution of the system E_i to the environmental chain stability. This is given by successive compositions of functors $\mathcal{H}_{i,i+1}^{(i)}$ $(i=1,2,\ldots,n)$.

Finally, and as a mean of describing the past stability of a given environmental structure such as E_{i+1}, it is possible to define a degree of contribution to stability of one system (in this case E_{i+1}) from previous ones, based on the number of subcategories $\mathscr{L}^{(i)}_{i+1}$.

D.1. The subcategories E^d_i and E^c_{i+1} are called domain and codomain subcategories of the respective categories E_i and E_{i+1}, when it is possible to define an isomorphic functor:

$$\mathscr{F}_{i,i+1} \;:\; E^d_i \longrightarrow E^c_{i+1} \qquad (3)$$

such that:

a) The cardinality of collections of objects $\{G_i\}$ and $\{G_{i+1}\}$ of E^d_i and E^c_{i+1} respectively, is maximum with respect to every other cardinality of every other pair of subcategories defined in similar form.

b) For any pair of objects $G_i^{\alpha_1}$, $G_i^{\alpha_2} \in E^d_i$ (and the respective objects $G^{\alpha_1}_{i+1}$, $G^{\alpha_2}_{i+1}$ identified with $\mathscr{F}_{i,i+1}$ on E^c_{i+1}); it must be that:

$$\left[G_i^{\alpha_1} , G_i^{\alpha_2}\right]_{E^d_i} = \left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i}$$

(and the same for:

$$\left[G^{\alpha_1}_{i+1}, G^{\alpha_2}_{i+1}\right]_{E^c_{i+1}} = \left[G^{\alpha_1}_{i+1}, G^{\alpha_2}_{i+1}\right]_{E_{i+1}} \quad)$$

but when the condition a) makes impossible to fulfill the foregoing, then:

$$\left[G_i^{\alpha_1} , G_i^{\alpha_2}\right]_{E^d_i} = \emptyset$$

although

$$\left[G_i^{\alpha_1} \quad G_i^{\alpha_2}\right]_{E_i} \neq \emptyset$$

(and similarly for:

$\left[G^{\alpha_1}_{i+1}, G^{\alpha_2}_{i+1}\right]_{E^c_{i+1}} = \emptyset$ although $\left[G^{\alpha_1}_{i+1}, G^{\alpha_2}_{i+1}\right]_{E_{i+1}} \neq \emptyset$)

Criteria a) and b) require further comentaries. The maximal cardinality condition established by a) may not imply a full subcategory for E_i^d and E_{i+1}^c. Thus, if it is given:

$$\mathfrak{F}_{i,i+1}\left(\left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i}\right) \neq \left[G_{i+1}^{\alpha_1}, G_{i+1}^{\alpha_2}\right]_{E_{i+1}}$$

then both morphism sets must be considered empty (condition b) within each respective <u>isomorphic</u> subcategories E_i^d and E_{i+1}^c.

D.2. Subcategory E_i^o of E_i is called <u>nucleus subcategory</u> when it is a subcategory of both E_i^d and E_i^c (domain and codomain subcategories of E_i) and it satisfies the following:

a) the cardinality of the collection of objects $\left\{G_i^o\right\}$ of E_i^o is maximum compared with other cardinalities of every other subcategories defined in a similar way.

b) for any pair of objects $G_i^{\alpha_1}, G_i^{\alpha_2} \in E_i^o$ it must be performed that:

$$\left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^o} = \left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^d} = \left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^c}$$

but, when the condition a) makes impossible to fulfill the foregoing, then;

$$\left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^o} = \emptyset \text{ although } \left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^d} \neq \emptyset$$

and/or

$$\left[G_i^{\alpha_1}, G_i^{\alpha_2}\right]_{E_i^c} \neq \emptyset$$

It is important to see, from the previous definition, that objects $G_i^{o'} \notin \left\{G_i^o\right\}$ of E_i^c do not satisfy $G_i^{o'} \in E_i^d$ and $G_i^{o'} \in E_i^c$, simultaneously.

D.3. The <u>full</u> subcategory E_i^i of E_i^d is called the <u>entering subcategory</u> when it is formed by a collection of objects $\{G_i^i\}$, such that:

a) The objects $G_i^i \in \{G_i^i\}$ are those objects of E_i^d fulfilling $G_i^i \notin E_i^o$.

As E_i^d may not be full of E_i, the same is true for E_i^i with respect to E_i.

D.4. The <u>full</u> subcategory E_i^e of E_i^c is called the <u>leaving subcategory</u> , when it is formed by a collection of objects $\{G_i^e\}$, such that:

a) the objects $G_i^e \in \{G_i^e\}$ are those objects of E_i^o such that $G_i^e \notin E_i^o$.

As E_i^c may not be full of E_i, the same is true for E_i^e with respect to E_i.

D.5. The subcategory $\mathcal{L}_i$ of E_i^i is called the <u>contributior subcategory</u> of E_i when an isomorphic functor

$$\mathcal{H}^{(i)}_{i,i+1} \ : \mathcal{L}_i \longrightarrow \mathcal{L}^{(i)}_{i+1} \qquad (4)$$

may be defined such that its codomain $\mathcal{L}^{(i)}_{i+1}$ is a <u>full</u> subcategory of the nucleus subcategory E^o_{i+1}.

a) Similar to D.2. a) with respect to collection of objects $\{G_i^k\}$ of $\mathcal{L}_i$.

From the diagram of Figure 2, it is shown that $\mathcal{L}_i$ may not be a full subcategory of E_i^i.

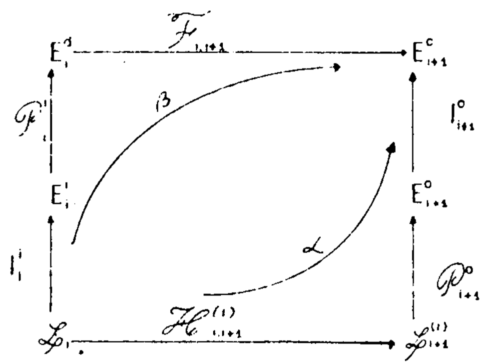


FIG. 2

As the isomorphic functor $\mathcal{F}_{i,i+1}$ and $\mathcal{H}^{(i)}_{i,i+1}$ imply fullness of these functors; the composition of full functors through the pathway α may be broken when the functor I^{o}_{i+1} is only a faithful functor. In this case, the composition of full functors through the pathway β , will be destroyed by the unique form given when I^{i}_{i} is only a faithful functor.

3.- STABILITY:

In order to have the global stability of an environmental system three specific concepts will be formulated. They are: the stability of a system the contribution of a system to the environmental chain stability, and the degree of contribution to stability in a system from previous ones.

As it was previously mentioned, the stability of an environmental system implies constancy of part of its structure along successive systems. This constancy is mathematically expressed by means of isomorphic functors placed on those parts of the structure (E_i^o and $\mathcal{L}_i$ subcategories) which remain similar along some successive environmental systems.

From a related point of view, these compositions must be understood as a simple mathematical concept, from which arises the temporal feasibility of the concept of stability. This can be drawn from the existing relationship between the double chain of successive biological and environmental systems, and the time axis.

In order to define the first and most important criterion of stability of an environmental system E_i, it is necessary to define the isomorphic functor

$$\mathcal{K}^{(i)}_{i+j-1,i+j} : E^o_{i+j-1} \longrightarrow E^{o(i+j-1)}_{i+j} \quad (5)$$

where $E^{o(i+j-1)}_{i+j}$ is a subcategory of E^o_{i+j}.

Given this isomorphic functor, it is defined that the stability S_i of an environmental system E_i has a value n ($S_i = n$) when:

1) the composition of n isomorphic functors

$$E_i^o \xrightarrow{\mathcal{K}^{(i)}_{i,i+1}} E_{i+1}^{o(i)} \xrightarrow{\mathcal{K}^{(i)}_{i+1,i+2}} E_{i+2}^{o(i)} \longrightarrow \cdots \xrightarrow{\mathcal{K}^{(i)}_{i+n-1,i+n}} E_{i+n}^{o(i)} \tag{6}$$

can be made; so that each subcategory $E_{i+j}^{o(i)}$ $(j=1,2,\dots,n)$ is a subcategory of its respective nucleus subcategories E_{i+j}^o and

2) Also if a new isomorphic functor:

$$\mathcal{K}^{(i)}_{i+n,i+n+1} : E_{i+n}^{o(i)} \longrightarrow E_{i+n+1}^{o(i)}$$

is defined (which may not be the case), then the $E_{i+n+1}^{o(i)}$ subcategory is not a subcategory of E_{i+n+1}^o.

This definition should be considered in more detail.

As it was remarked earlier with reference to the diagram in Figure 1, the nucleus subcategory E_i^o represents the maximum structure whose initial conformation has been determined in the system E_{i-1} (as part of the structure of E_{i-1}^d). Moreover, E_i^c also exists in the system E_{i+1} (as part of the structure of E_{i+1}^c) by functor $\mathcal{F}_{i,i+1}$. Thus, when the original nucleus subcategory E_i^o can be transferred from the original system to other n nucleus subcategories of subsequent systems, the former's <u>stability</u> is determined by n.

Another criterion of stability evolves immediately from the isomorphic functor

$$\mathcal{H}^{(i+j-1)}_{i+j-1,i+j} : \mathcal{L}_{i+j-1} \longrightarrow \mathcal{L}^{(i+j-1)}_{i+j}$$

Thus, it is stated that the contribution $\mathcal{A}_i$ of the E_i system to the environmental chain stability is n ($\mathcal{A}_i = n$), when the composition of only n isomorphic functors like

$$\mathcal{L}_i \xrightarrow{\mathcal{H}^{(i)}_{i,i+1}} \mathcal{L}^{(i)}_{i+1} \xrightarrow{\mathcal{H}^{(i)}_{i+1,i+2}} \mathcal{L}^{(i)}_{i+2} \longrightarrow \cdots \xrightarrow{\mathcal{H}^{(i)}_{i+n-1,i+n}} \mathcal{L}^{(i)}_{i+n} \qquad (7)$$

can be made.

The above definition takes into consideration the number of times that the contribution subcategory $\mathcal{L}_i$ of E_i is included as a full subcategory of subsequent nucleus subcategories E^o_{i+j} ($j = 1,2,\ldots$). As these determine the stability S_{i+j} of the successive systems, and as the original contribution subcategory $\mathcal{L}_i$ is formalized by means of new objects and morphisms entering E_i, and by the chain of environmental systems subsequent to E_i, the evidence for a second criterion of stability is therefore readily obtained.

With respect to the degree of contribution to stability in a system E_{i+s} from previous ones, it can be stated that if for a given nucleus subcategory E^r_{i+s}, p subcategories $\mathcal{L}^{(\alpha)}_{i+s}$ ($\alpha = i,j,k,\ldots$) of E^o_{i+s} are defined, such that they are given by the composition of isomorphic functors $\mathcal{H}$, then the degree of contribution $\mathcal{G}_{i+s}$ in the E_{i+s} system is p ($\mathcal{G}_{i+s} = p$).

In fact, the p contribution subcategories $\mathcal{L}^{(\alpha)}_{i+s}$ do not have to correspond to original successive environmental systems, and moreover, each $\mathcal{L}^{(\alpha)}_{i+s}$ may not be a full subcategory of E^o_{i+s}, because the respective $E^{o(\alpha)}_{i+s}$ may not be full subcategories of E^o_{i+s} ($\alpha \neq i+s-1$).

This criterion $\mathcal{G}_{i+s}$ is concerned with the past of stability of the system for which it is defined. It comprises the entire history of a certain part of the structure of the system E_{i+s} (E^{o}_{i+s}) where the stability S_{i+s} will be defined.

The significance of this criterion must not be underestimated by considering it as merely derived from the other criteria. Thus, the relative age of each $\mathcal{L}^{(\alpha)}_{i+s}$, the cardinality of their collection of objects, and the constancy and number of morphisms from the previous structures $\mathcal{L}_{\alpha}$ to E^{o}_{i+s}, are enough evidence about the usefulness of this concept. In fact, if the chain of environmental systems is regarded as due to the chain of biological systems, the later performance is directly obtained from $\mathcal{G}_{i+s}$.

4.- ON MORPHISMS AND OBJECTS IN AN ENVIRONMENTAL CATEGORY.

Let us define the full subcategory X_i of E_i, so that its collection of objects $\{G_x\}$ will be determined by objects $G_x \in E_i$ with $G_x \notin E^{d}_{i}$, $G_x \notin E^{c}_{i}$.

With respect to morphisms, it must be noted that certain morphisms $g \in [G_1, G_2]_{E_i}$ may exist such that $g \notin E^{d}_{i}$, $g \notin E^{c}_{i}$, $g \notin X_i$.

This is the case, when $g \in [G_1, G_2]_{E_i}$ with $G_1, G_2 \in E^{d}_{i}$ (or $G_1, G_2 \in E^{c}_{i}$) but $g \notin E^{d}_{i}$ (or $g \notin E^{c}_{i}$).

The same is true when the objects G_1, G_2 are not objects of the same subcategory E^{d}_{i}, E^{c}_{i} and X_i; for instance $G_1 \in E^{d}_{i}$ and $G_2 \in X_i$.

From another point of view, let us define as <u>endo-objects</u> of an object $G_i^{\alpha} \in E_i$, the objects $G_{i-j}^{\alpha} \in E_{i-j}$ $(j = 1,2,\dots,m)$ and $G_{i+k}^{\alpha} \in E_{i+k}$ $(k=1,2,\dots,n)$, so that these last objects are identically defined as the objects G_i^{α}.

The definition yields the following very simple theorems:

T1) Given a succession of environmental cateogories

$$\dots \longrightarrow E_{i-1} \longrightarrow E_i \longrightarrow E_{i+1} \longrightarrow \dots$$

if $G_i^{\alpha} \in I_i$; then endo-objects $G_{i-1}^{\alpha} \in E_{i-1}$ and $G_{i+1}^{\alpha} \in E_{i+1}$ of G_i^{α} will not exist.

<u>PROOF</u>: It is immediate.

If $G_i^{\alpha} \in I_i \Rightarrow G_i^{\alpha} \notin E_i^d$ and $G_i \notin E_i^c$

and from the functors

$$\mathcal{F}_{i,i+1} : E_i^d \longrightarrow E_{i+1}^c$$

$$\mathcal{F}_{i-1,i} : E_{i-1}^d \longrightarrow E_i^c$$

and from the definiticn of E_i^d and E_i^c, it follows that $G_{i-1}^{\alpha} \notin E_{i-1}^d, \notin E_{i-1}$ and $G_{i+1}^{\alpha} \notin E_{i+1}^c, \notin E_{i+1}$.

T2) Given a succession of environmental categories

$$\dots \longrightarrow E_{i-1} \longrightarrow E_i \longrightarrow E_{i+1} \longrightarrow \dots$$

if $G_i^{\alpha} \in E_i^o$

a) either $G_{i-1}^{\alpha} \in E_{i-1}^o$ or $G_{i-1}^{\alpha} \in E_{i-1}^i$ and $G_{i-1}^{\alpha} \in \mathcal{L}_{i-1}$

b) either $G^{\alpha}_{i+1} \in E^{o}_{i+1}$ or $G^{\alpha}_{i+1} \in E^{e}_{i+1}$.

<u>PROOF</u>: It is immediate.

T3) Given a succession of environmental categories

$$\ldots \longrightarrow E_{i-1} \longrightarrow E_i \longrightarrow E_{i+1} \longrightarrow \ldots$$

if $G^{\alpha}_i \in E^{e}_i \Rightarrow$

a) either $G^{\alpha}_{i-1} \in E^{o}_{i-1}$ (from T2) or $G^{\alpha}_{i-1} \in E^{i}_{i-1}$ and $G^{\alpha}_{i-1} \notin \mathcal{L}_{i-1}$

b) $G^{\alpha}_{i+1} \notin E_{i+1}$

<u>PROOF</u>: It is interesting to prove that if $G^{\alpha}_i \in E^{e}_i \Rightarrow$ $\Rightarrow G^{\alpha}_{i+1} \notin E_{i+1}$.

As $G^{\alpha}_i \notin E^{o}_i \Rightarrow G^{\alpha}_i \notin E^{d}_i$, then the possible endo-object G^{α}_{i+1} of G^{α}_i, will satisfy $G^{\alpha}_{i+1} \notin E^{c}_{i+1}$.

Moreover, in the same fashion, from T1 $G^{\alpha}_{i+1} \notin X_{i+1}$.

Then, as it is also true that $G^{\alpha}_{i+1} \notin E^{d}_{i+1}$ then $G^{\alpha}_{i+1} \notin E^{d}_{i+1}, \notin E^{c}_{i+1}, \notin X_{i+1} \Rightarrow G^{\alpha}_{i+1} \notin P_{i+1}$.

T4) Given a succession of environmental categories

$$\ldots \longrightarrow E_{i-1} \longrightarrow E_i \longrightarrow E_{i+1} \longrightarrow \ldots$$

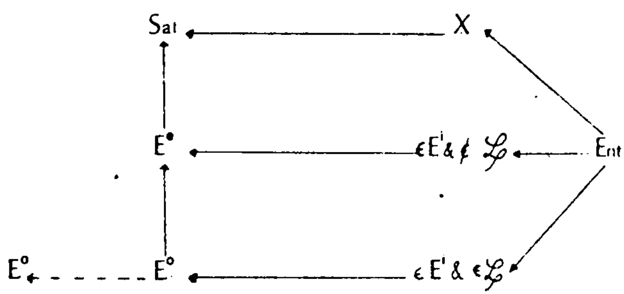
if $G^{\alpha}_i \in E^{i}_i$ and $G^{\alpha}_i \in \mathcal{P}_i \Rightarrow$

a) $G^{\alpha}_{i-1} \notin E_{i-1}$

b) $G^{\alpha}_{i+1} \in E^{o}_{i+1}$ (from T2)

<u>PROOF</u>: It is immediate.

These simple theorems are useful to describe the object's dynamics through any succession of environmental categories. Thus, we can represent (Figure 3) these results in a graph whose nodes are subcategories, and whose oriented edges represent the objects succession through the environmental chain.



Ent = entrance

Sal = exit

FIG. 3

From this graph, it follows that;

- There are no cycles.
- Not every object entering an environmental chain contributes to stability.

- The exit nodes (or subcategories) are E^e and X.
- If an object C_i^{α} contributes to the stability of E_i, its endo-object G_{i-1}^{α} may contribute to the environmental chain stability.

5.- DISCUSSION:

Structural changes of E or M resulting of their interactions, can be used as account for the consequences verified in one system when the other is modified. Therefore, the definition of environmental stability arises naturally and constitutes a new mean of measuring the behavior of M (and of E itself).

As it was noted earlier, the criterion of stability implies that a certain part of the structure of an original system E_i remains constant along successive systems subsequent to it. It must be recognized, how the theory of category, by which the environment was represented, was found to be useful to represent these certain structural parts and the constancy referred above, by means of subcategories and by a composition of n similar isomorphic functors, respectively. Moreover, the variation of time is naturally included in the criterion of stability because there exists a correspondence between the composition of n isomorphic functors ($\mathcal{K}^{(i)}_{i,i+j}$ or $\mathcal{H}^{(i)}_{i,i+j}$), and the time axis.

The preceding analysis shows how the global stability of an environmental system is related to its previous history and projected to its future. The degree of contribution g represents a measure of the "past of stability" while the contribution (h) of a system to the environmental chain stability indicates the future, in relation to the partial data of stability for the subsequent systems for which h is defined. The general data of stability of each system E_i is given by the concept that defines S_i, by means of which the constancy of certain structural part of E_i is recognized along a part of the environmental chain.

The new mathematical interpretation of the concept of stability requires a simple comment. It has been introduced as a mean of recognizing a repetition of basic structures occurring within greater structures of systems which follow a successional pattern.

As the mathematical concepts involved in these new criteria regard, it is to be hoped that new ones will be introduced provided they are consistent with the biological and environmental considerations on which this work has been baced.

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PLACE DES ETRES ORGANISES

SUR L'ECHELLE DE TEMPS

(8ème Congrés International de Biologie Mathématique sept.1981) par Charles BOGDANSKI ,Professeur associé de Biophysique à l'université René Descarte (Paris).

SUMMARY

PLACE OF THE LIVING BENGS ON THE SCALE OF TIME

This text constitues a new element in a present author's series of articles in the Revue de BIOMATHEMATIQUE concerning the dimensional formal treatment of Living Beings,considered as one of the types family of the natural self-regulatory systems.
This type remains marginal for the so-called *"classical"* Physics,due to the fact that the complexity - from the viewpoints of architecture,struture and shape - of the representatives of the Biotic type,lends itself poorly to formal treatment as it is usually practiced with respect to inanimate systems,the systems called *"physical"*.
Having presented previosly essential size aspects of the dimensional treatment of Living Beings,the present author is starting now with an initiation to the time aspects and particulary to the interdependence aspect of some values concerning Biotic Systems.