European Journal of Physics

Mean free path in soccer and gases

To cite this article: J Luzuriaga 2010 *Eur. J. Phys.* **31** 1071

IOP Publishing European Journal of Physics

Eur. J. Phys. **31** (2010) 1071–1076 doi:10.1088/0143-0807/31/5/009

Mean free path in soccer and gases

J Luzuriaga

Centro Atómico Bariloche- CNEA, Instituto Balseiro UNC (8400), Bariloche, Argentina

E-mail: luzuriag@cab.cnea.gov.ar

Received 12 April 2010, in final form 23 June 2010
Published 21 July 2010
Online at stacks.iop.org/EJP/31/1071

Abstract
The trajectories of the molecules in an ideal gas and of the ball in a soccer game are compared. The great difference between these motions and some similarities are discussed. This example could be suitable for discussing many concepts in kinetic theory in a way that can be pictured by students for getting a more intuitive understanding. It could be suitable for an introductory course in vacuum techniques or undergraduate courses in kinetic theory of gases. Without going into the slightly harder quantitative results, the analysis presented might be used for introducing some ideas of kinetic theory qualitatively to high school students.

Molecular motion is present all around us, yet we have no direct perception of it. We learn that heat is due to the kinetic energy of molecular motion, but the only clue we can have through our senses is indirect through this sensation of heat. We have to use our imagination to visualize the motion of atoms and molecules going on around us all the time. The description in terms of the kinetic theory of heat is necessarily abstract and remote from direct experience. When teaching vacuum techniques to both graduates and undergraduates, the mean free path is an important concept to understand because it determines the flow regime. However, it is rather abstract and the different formulae being applied to the molecular or viscous regimes may appear arbitrary to those students that find the idea of the mean free path hard to understand.

A comparison of the movement of objects of macroscopic dimensions, which we experience daily, and those at the microscopic scale, could help to appreciate the important differences, as well as some similarities that are due mainly to chance. In the game of soccer we can find one of those chance similarities, as is shown below. Soccer is familiar to many students and can be used as a starting point to visualize the differences between the molecular motions in a dilute gas and motion in everyday experience.

We start by considering an ideal gas [1][1] and compare the motion of gas molecules to that of footballs.

In gases, as well as in soccer, atoms or molecules spend a large portion of time in almost free flight, until they hit another atom or a player's foot. The distance of free flight is the mean

---

[1] The reference is the edition consulted by the author and has a good discussion. Newer editions of this same work are also available.

free path of gases, although it has no name in soccer. There is by coincidence, however, a similarity of scales between the size of atoms and the mean free path, and the size of footballs and the length of a more or less standard kick, as is shown below.

There is an approximate expression that is a rather good practical approximation for the mean free path λ.[2]. In air at room temperature for a given pressure P, λ is given by

$$\lambda(\text{cm}) = \frac{5 \times 10^{-3}}{P(\text{torr})} \tag{1}$$

or equivalently

$$\lambda(\text{mm}) = \frac{6.6}{P(\text{Pa})}; \tag{2}$$

the mean free path is inversely proportional to pressure, and we have included two popular pressure units for convenience.

As a rough approximation the diameter d_{mol} of a nitrogen or oxygen molecule can be taken[3] as $d_{\text{mol}} = 0.35$ nm, assuming an average diameter to represent the mixture of elements present in air. The result of calculating the mean free path l, at room temperature and normal pressure, is $l \simeq 0.066\ \mu\text{m} = 66$ nm. Therefore the ratio $l/d_{\text{mol}} \simeq 200$.

A soccer ball, on the other hand, has a diameter of 25 cm = 0.25 m, that is, about nine orders of magnitude greater than a gas molecule. Therefore, if we blew up the molecules to soccer ball proportions, the corresponding mean free path would be $200 \times 0.25 = 50$ m. This is about the length of half the football pitch. Some goalkeepers routinely kick the ball further than this, and short passes are considerably shorter. Therefore, if we look at the trajectories of the ball in a football game and try to make abstraction of the players, the free flight of the ball would look similar to that of the atoms in air. We are assuming that, on average, the free distance the football will go before being kicked by a player would be around 40 m (in a game dominated by long kicks, to be sure). As in the gas case, it is also possible to find much shorter and much longer free flights.

So, if it were possible to look at individual molecules and follow their paths, the trajectories would be in some sense similar to that of a soccer ball in a match. Of course, in one case there would be a multitude of other molecules, which we ignore, and in a football game we must ignore the players. However, this is precisely the sort of abstraction made in statistical mechanics calculations for the mean free path. A rough visualization is given in figure 1. In spite of the similarity in the ratio of mean free path to diameter, the differences between the movement of molecules and soccer balls are very great.

In the first place, speeds are enormously different.

The average velocity for molecules (assuming room temperature) is around 400 m s^{-1}, of the same order as the speed of sound. Footballs go much slower. The maximum speed is around 30 m s^{-1} (100 km h^{-1} approximately). A soccer match speeded up to molecular velocities would go fast indeed.

The high speeds and small distances in a gas imply very frequent collisions between the molecules. The average time between collisions τ is roughly

$$\tau = \frac{66 \times 10^{-9}\ \text{m}}{400\ \text{m s}^{-1}} = 1.6 \times 10^{-10}\ \text{s}, \tag{3}$$

---

[2] An interactive calculator for the mean free path is found at http://230nsc1.phy-astr.gsu.edu/hbase/kinetic/menfre.html as well as a derivation of the formulae at http://230nsc1.phy-astr.gsu.edu/hbase/kinetic/menfre.html.

[3] This is a conventional value, which would give a mean free path compatible with the simple approximate formula given in the text and the online calculator mentioned in the preceding footnote. Molecular or atomic dimensions do not have a unique experimental value, an idealization such as considering them as hard spheres will give a different value than obtained with x-rays in different compounds, etc. A discussion is beyond the scope of this paper, but it should be kept in mind when presenting the idea of a hard sphere ideal gas.

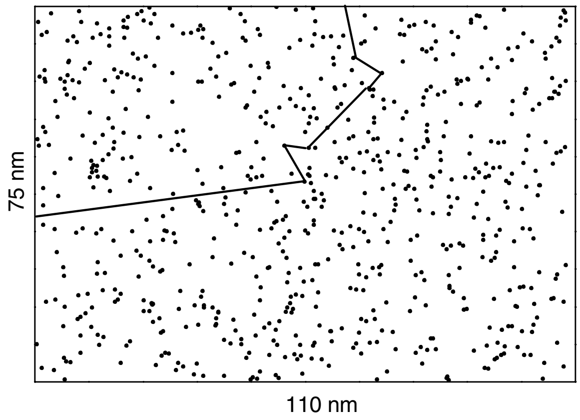

Figure 1. A soccer field reduced to molecular dimensions, showing random positions of molecules. The mean distance between molecules is much smaller than the mean free path. A possible trajectory is shown, starting approximately from the goal position and escaping through a side after a few collisions. Because the dimensions of the area considered are comparable to the mean free path, many trajectories are expected to hit the walls. In soccer the ball often leaves by the sides, but since this is an involuntary move by players, it is probable that differences could conceivably be found in the frequency of 'wall hits' in soccer and gases.

an extremely short time. In contrast, the average time between collisions in a soccer match can lay between a few seconds to fractions of a second. Time intervals of the order of the time between collisions in a gas are not only beyond the reach of our senses but are also hard to measure using fast electronics.

Another difference between gases and football is apparent when we consider the force of gravity, which is an important factor in soccer. In football the movement is almost two dimensional, in the sense that the ball is hitting the ground as often as the players, and the height above the field is only a fraction of the free path in most shots. We may try for a closer analogy by considering the case of gases adsorbed in a two-dimensional substrate, but the molecules are attracted to the surface by the forces of van der Waals type, and they do not have the same distance dependence as the force of gravity. Because the van der Waals forces are of much shorter range, there is no proper molecular analogy for the 'quasi-2D' behaviour of the soccer ball in the pitch.

In both cases, molecules and soccer ball, the action is almost non-stop. However, it is important to point out that there is a big difference in the energy balance in both examples. In the case of the soccer ball, it is the players who supply the energy for motion with their kicks, while friction with air and ground tends to stop the ball. In the simplified version of a gas used here, there is no energy loss in collisions, so energy is conserved in each individual collision.

There are two possibilities that have to be considered for the overall energy present in the gas. Isothermal conditions may be specified, where the gas is considered to be in contact with a thermal reservoir. This is generally the case in an ordinary room or container. Here, energy is continually exchanged with the reservoir, usually through the walls, and molecules interacting at the wall are the ones that maintain the balance of energy on average. In each collision with the wall energy might be given or removed from the thermal bath and there are fluctuations.

If adiabatic conditions are specified, either perfectly elastic or perfectly rigid walls can be postulated, so that energy is conserved in each rebound, and the total energy is strictly conserved in each collision, including walls. The rigid or perfectly elastic walls do not allow thermal fluctuations globally, but because of the energy exchange between molecules that collide, there are energy fluctuations in the kinetic energy of a molecule and also in subregions inside the container. There would be little difference in the mean free path in either case, except perhaps near the walls.

In soccer, the boundary conditions are more open. The ball leaves the field occasionally and is thrown back in after a short time (although in professional matches it could be a different, almost identical ball). This is closer to an isothermal regime because the energy with which the ball leaves the field may be greater or smaller than that with which it is thrown in. And there is also an analogy with the temporary absorption of atoms on a surface for the molecular case, where a molecule may spend some time adsorbed on the wall and return to the gas later. The open boundary conditions, and the ‘energy exchange’ at the walls, make soccer a closer analogy with the isothermal case in a gas. Of course the ‘average energy’ of the ball is independent of a thermal reservoir and depends on the energy supplied by the players. Here a weak analogy exists with temperature. We may consider a professional match, with more energy per kick, and an amateur or junior game. In the first case, the average temperature is higher, in the second lower.

One question that is not properly a physics question but may be interesting to contemplate is that of the will of the players. Surely, in soccer, passes are not made at random (or at least, not very often). However, the player positions change, in a pattern that is difficult to predict, and therefore the ball is directed to these ‘random’ positions. It may be interesting to debate whether it is possible to distinguish this pseudo-random movement from the truly random motions of molecules. In fact, the positions of the players change because players from one team try to avoid those of the other, but the many factors that come into the decision to pass or the loss of the ball complicate matters a lot. When looking at velocities both situations are much easier to distinguish because of the conservation of kinetic energy in a gas collision and the lack of it in soccer, as mentioned before. For example, the probability for a molecule to come to rest after a collision is small because the collision has to be head on and with a molecule of the same speed, but in soccer, players routinely stop a long pass absorbing the energy of the ball, and so large reductions of speed are frequent.

However, if the information available is only the trajectories, without clues about the time elapsed, differences could still be detected. To put the question in a more concrete form we could ask: Presented with a picture of ball trajectories in a soccer game and the printout of a molecular simulation of gas trajectories in a ‘hard sphere’ gas, would we be able to detect any difference?

In fine detail, things are not the same, certainly, but detecting a difference would perhaps need a quite detailed statistical analysis. The statistics of the ‘mean free path’ in soccer, that is the statistical distribution of kick lengths, is not easy to come by. Although some companies produce statistical data for professional soccer clubs and trainers[4], they apparently concentrate on other data, and they are available only for a fee. However, other quantities besides the magnitude of the mean free path are probably much better at detecting a difference. Angular correlations in the change of direction would almost certainly give a large discrepancy. In the centre of mass system, the angular dispersion of hard sphere collisions is uniform, that is, any angle is equally probable. In a soccer match, both teams have a preference for moving the ball in the direction of the opposite goal, and this bias may be evident. There is an even stronger bias for keeping the ball inside the field because in this both teams have the same interest. This makes the ball remain within the boundaries for a longer time than in an equivalent random

---

[4] Companies that provide data for professionals can be found at: http://213.30.139.108/sport-universal/uk/amiscopro.htm or http://www.prozonesports.com/.

problem, as discussed below. In fact, the pitch size (the container) is of the same order of magnitude as the mean kick length (mean free path), a situation that in a gas is usually termed the molecular regime [2], where a molecule has a probability of hitting a wall comparable to that of colliding with another molecule. The probability of hitting a wall in a gas can be deduced from the number of molecules hitting a wall per second, which as calculated in standard texts such as Dushman [2] is 3×10^{-23} s^{-1} cm^{-2}. This is for a three-dimensional system, and to adapt to our almost two-dimensional case, we arbitrarily take a distance of 15 m for the vertical dimension of the football pitch, and this is what we scale to molecular dimensions. Therefore we are left with a micro field of size 110 × 75 × 15 nm. The lateral area (perimeter by height) comes out as 5×10^3 nm^2 and the number of collisions is 1.6×10^{13} s^{-1}. The number of molecules inside the volume, at normal temperature and pressure, is approximately 3.3×10^3; therefore, each molecule is expected to strike a wall about once every 2×10^{-10} s. This is almost equal to the collision time τ between molecules, estimated above from the mean free path and average speed. This means it is equally likely for a gas molecule to hit a wall or another molecule. In soccer however, several passes are completed before the ball is sent outside the field. A rough estimate could be made by students in any match; my guess is around a dozen at least. Therefore, just by looking at trajectories and seeing in which case the walls do not tend to be hit, one could distinguish a soccer match from a molecular simulation.

An increase in the ratio of mean free path to pitch length so that we change to the viscous regime [2] in a soccer game would need a change in dimensions of at least a factor of a hundred. This would give a soccer field which is around 10 × 7 km if scaled up. The prospect of playing in such a field is daunting for anyone who has the experience of a game on a normal field, but it may give a feeling for the physical difference in both regimes. The players would need around a hundred long passes to reach a border and a fairly long time would elapse too. For visualizing the intermediate regime [2], a field measuring a few city blocks (New York style, for example) can be imagined. This scaling up could perhaps make it easier for students to get a feeling of why we need different formulae (e.g. for impedance and pumping speeds) and why we observe different behaviour in each regime.

It could also be interesting to compare the mean free path for a couple of pressures common in many vacuum systems and to use the same artefact of scaling molecule and distances to football proportions. At 10^{-2} torr, a vacuum corresponding to a mechanical pump, the mean free path is $l = 0.5$ cm. If the molecule were as big as a soccer ball, it would fly around 3500 km before hitting the next molecule, that is, it would cross the United States from coast to coast relatively frequently. The molecular regime in this scaled-up case would happen in a space with the dimensions of a continent. At 10^{-6} torr, a pressure which can be reached with a mechanical/diffusion pump combination, the scaled-up mean free path is four orders of magnitude greater, that is, 3×10^8 km, about twice the Earth to Sun distance of 150 million km. Perhaps the scaled-up example could also be used for helping students to get an intuitive grasp of the difference between a change in regime and a phase transition. The molecular regime is reached for larger containers at a lower pressure, as illustrated above, while a phase transition such as the familiar gas–liquid transformation of water at 100 °C takes place when the pressure is above 1 atmosphere, independent of the size of the container.

We hope this comparison between a popular sport such as soccer and the ideal gas model might be a way of introducing some of the concepts of kinetic theory of gases to students, perhaps allowing easier visualization and understanding of some ideas in a more intuitive, less purely abstract way. In particular, the ratio of mean free path to dimensions of pipes or containers, that determines the molecular and viscous regimes, is more easily pictured when scaling up the dimensions of molecules. In a vacuum course or when designing a vacuum system, this intuition may help students to select the appropriate formulae for each case.

References

[1] Sears F W and Zemansky M W 1953 *University Physics* (Reading, MA: Addison-Wesley)
[2] Dushman S 1962 *Scientific Foundations of Vacuum Technique* 2nd edn (New York: Wiley) p 14