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Observation on the Weak Nordheim Rule and on Isomerism in Odd–Odd Nuclei

By C. A. Mallmann*

NORDHEIM RULES

Nordheim's rules[1] have been compared recently by K. Way *et al.*[2] with experimental results for medium weight nuclei ($21 \leqslant N \leqslant 81$).

The odd-A nuclei in this region were divided into groups, limited by proton or neutron numbers, in which only certain ground state spin values are observed. The limits of these groups are not very sharp. This is the reason why in a group one sometimes observes a ground state spin value which should appear in a neighboring group.

These same proton and neutron numbers were used to define groups of odd–odd nuclei in which the experimental ground state spin values were compared with those predicted by Nordheim's rules, using all possible combinations of the allowed ground state spin values for protons and neutrons in the corresponding odd-A groups. Again one has to note that the limits of these odd–odd groups are not very sharp.

Using this criterion, one obtains the spins of Table 1 and may verify that of 81 cases, 44 are explained by the strong rule, 21 by the weak rule, 1 by either the strong or the weak rule, 3 by the weak rule or by being exceptions, and 12 are exceptions.

The agreement with experiment is better if one changes the weak Nordheim rule

$$|j_\pi - j_\nu| < J \leqslant j_\pi + j_\nu \quad \text{if } j_\pi + j_\nu + l_\pi + l_\nu = \text{odd},$$

for the following one

$$|j_\pi - j_\nu| \leqslant J \leqslant j_\pi + j_\nu \quad \text{if } j_\pi + j_\nu + l_\pi + l_\nu = \text{odd}.$$

With this rule, which only means that one may use j, j coupling, one predicts the spins given by the old rule plus those given in parentheses in Table 1. Of the 81 cases one then gets 44 which are explained by the strong rule; 31 by the weak rule; 2 in which one may apply the strong or the weak rule (Sb^{116}, Cu^{60}); and 4 exceptions (K^{40}, In^{114}, In^{116}, In^{118}). This agreement may be considered good, taking into account the fact that the limits of the groups are not very sharp.

In Table 2 we have given the number of cases in which the spin values predicted by the weak rule are observed. In cases in which there are two possible experimental spin assignments each of them is indicated as $\frac{1}{2}$. From the figures one deduces that there seems to be no preference for any of the predicted spin values.†

The preference for high spin values which was observed by Nordheim may have been due to the fact that higher spin isomers are easier to detect experimentally.

ISOMERISM IN ODD–ODD NUCLEI

We have seen that within the groups in which we have classified the odd–odd nuclei every spin predicted by Nordheim's rules is a possible ground state spin. This shows that these spin states have energies which may be quite close, and as some of them have very different spin values they may be the cause of long-lived isomers. Looking for spin differences larger or equal to 3 one finds three different types of possible long-lived isomers:

(*a*) Those in which the initial and final configurations are the same and only a recoupling takes place. This is only possible if the weak Nordheim rule applies. According to D. E. Alburger's review[3] on nuclear isomers, Sc^{44}, Sc^{46}, Mn^{46}, Co^{58} and Co^{60} could be examples of this possibility. These isomers have no change of parity.

(*b*) Those in which the configuration of the initial state is different from that of the final state with a change of level of one particle. We may distinguish two cases:

(1) Going from a state in which one of the unpaired particles has a spin parallel to its orbital angular momentum and the other antiparallel (S.R.), to a state in which both unpaired particles have spins parallel or antiparallel to their orbital angular momenta (W.R.). These states may have the same parity; Rb^{86}, Br^{80} and In^{112} could be examples[3] of this case. They may also have different parity because the unpaired particle which changes

* Comisión Nacional de Energía Atómica.

† After this article was written, S. A. Moszkowski kindly sent me a preprint of his article with C. J. Gallagher Jr. on the coupling of angular momenta in odd–odd nuclei. He gives theoretical arguments for the "breakdown" of the weak Nordheim rule and also shows in his discussion of empirical data that the rule is "very much weaker than previously thought".

Table 1[a]

π[b]	ν[c]	*Strong rule*		*Weak rule*	
21–27	21–27			(π1f7/2)(ν1f7/2)	(0), 1, 2, 3, 4, 5, 6, 7+
21–27	29–37	(π1f7/2)(ν1f5/2)	1+	(π1f7/2)(ν2p3/2)	(2), 3, 4, 5+
29–37	29–37	(π2p3/2)(ν1f5/2)	1+	(π2p3/2)(ν2p3/2)	(0), 1, 2, 3+
		(π1f5/2)(π2p3/2)	1+	(π1f5/2)(ν1f5/2)	(0), 1, 2, 3, 4, 5+
29–37	39–49	(π2p3/2)(ν2p1/2)	1+	(π2p3/2)(ν1g9/2)	(3), 4, 5, 6−
		(π1f5/2)(ν1g9/2)	2−	(π1f5/2)(ν2p1/2)	(2), 3+
29–37	51–63	(π2p3/2)(ν1g7/2)	2−	(π2p3/2)(ν2d5/2)	(1), 2, 3, 4−
		(π1f5/2)(ν2d5/2)	0−	(π1f5/2)(ν1g7/2)	(1), 2, 3, 4, 5, 6−
39–49	39–49	(π2p1/2)(ν1g9/2)	4−	(π2p1/2)(ν2p1/2)	(0), 1+
		(π1g9/2)(ν2p1/2)	4−	(π1g9/2)(ν1g9/2)	(0), 1, 2, 3, 4, 5, 6, 7, 8, 9+
39–49	51–63	(π1g9/2)(ν1g7/2)	1+	(π1g9/2)(ν2d5/2)	(2), 3, 4, 5, 6, 7+
		(π2p1/2)(ν2d5/2)	2−	(π2p1/2)(ν1g7/2)	(3), 4−
39–49	65–81	(π2p1/2)(ν1h11/2)	5+	(π1g9/2)(ν1h11/2)	(1), 2, 3, 4, 5, 6, 7, 8, 9, 10−
		(π1g9/2)(ν2d3/2)	3+	(π2p1/2)(ν2d3/2)	(1), 2−
				(π2p1/2)(ν3s1/2)	(0), 1−
				(π1g9/2)(ν3s1/2)	(4), 5+
51–63	65–81	(π1g7/2)(ν1h11/2)	2−	(π1g7/2)(ν2d3/2)	(2), 3, 4, 5+
		(π2d5/2)(ν2d3/2)	1+	(π2d5/2)(ν1h11/2)	(3), 4, 5, 6, 7, 8−
				(π1g7/2)(ν3s1/2)	(3), 4+
				(π2d5/2)(ν3s1/2)	(2), 3+

[a] A level is given by (*n*, *l*, *j*).
[b] π—proton.
[c] ν—neutron.

Table 2[a]

π	ν	*Weak rule spins and experimental cases observed*		*Nuclei*
21–27	21–27	(π1f7/2)(ν1f7/2) spin	0, 1, 2, 3, 4, 5, 6, 7+	Sc^{44}, Sc^{46}, Sc^{48},
		No. of cases	3, $\frac{1}{2}$, $\frac{1}{2}$, 0, 2, 1, 1.5, 0.5	V^{46} V^{48}, V^{50}
				Mn^{50}, Mn^{52} Co^{54}
21–27	29–37	(π1f7/2)(ν2p3/2) spin	2, 3, 4, 5+	Sc^{50}, V^{52}
		No. of cases	3.5, 0.5, 1.5, 1.5	Mn^{54}, Mn^{56}
				Co^{58}, Co^{56}, Co^{60}
29–37	29–37	(π1f5/2)(ν1f5/2) spin	0, 1, 2, 3, 4, 5+	
		(π2p3/2)(ν2p3/2) spin	0, 1, 2, 3+	
		No. of cases	0, 0.5, 0.5, 0, 0.5, 0.5	As^{70}
29–37	39–49	(π1f5/2)(ν1p1/2) spin	2, 3+	
		No. of cases	0, 0	
		(π2p3/2)(ν1g9/2) spin	3, 4, 5, 6−	Ga^{72}
		No. of cases	0.5, 0.5, 0, 1	Br^{82}
29–37	51–63	(π1f5/2)(ν1g7/2) spin	1, 2, 3, 4, 5, 6−	
		(π2p3/2)(ν2d5/2) spin	1, 2, 3, 4−	
		No. of cases	0, 0, 0, 0, 0, 0	
39–49	39–49	(π1g9/2)(ν1g9/2) spin	0, 1, 2, 3, 4, 5, 6, 7, 8, 9+	
		(π2p1/2)(ν2p1/2) spin	0, 1+	
		No. of cases	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	
39–49	51–63	(π1g9/2)(ν2d5/2) spin	2, 3, 4, ·5, ·6, ·7+	Nb^{92}, Nb^{94}, Te^{94},
		No. of cases	2, 2, 0, 0·5, 1, 0·5	Tc^{96}, Rh^{98}, In^{110}
		(π2p1/2)(ν1g7/2) spin	3, 4−	
		No. of cases	0, 0	
39–49	65–81	(π1g9/2)(ν3s1/2) spin	4, 5+	
		No. of cases	0, 0	
		(π1g9/2)(ν1h11/2) spin	1, 2, 3, 4, 5, 6, 7, 8, 9, 10—	
		(π2p1/2)(ν3s1/2) spin	0, 1−	Ag^{112}
		(π2p1/2)(ν2d3/2) spin	1, 2−	
		No. of cases	0, 1, 0, – – – – – – –, 0	
51–63	65–81	(π1g7/2)(ν2d3/2) spin	2, 3, 4, 5+	Cs^{134}
		(π1g7/2)(ν3s1/2) spin	3, 4+	Cs^{136}
		(π2d5/2)(ν3s1/2) spin	2, 3+	La^{138}
		No. of cases	0, 0, 1.5, 1.5	
		(π2d5/2)(ν1h11/2) spin	3, 4, 5, 6, 7, 8−	Sb^{124}
		No. of cases	1, 0, 0, 1, 0, 0	I^{130}

[a] The symbols have the same meaning as in Table 1.

its level is near to the closure of a shell; Rh^{104} and Sb^{124} could be examples[3] of this case.

(2) Going from a state in which both unpaired particles have their spins parallel or antiparallel to their orbital angular momenta (W.R.), to another state in which also both unpaired particles have their spins parallel or antiparallel to their orbital angular momenta (W.R.). Also here there exists the possibility of having change in parity or not.

(*c*) Those in which the configuration of the initial state is different from that of the final state with a change of level of two particles. We may again distinguish two cases:

(1) Going from a state in which one of the unpaired particles has the spin parallel to its orbital angular momentum and the other antiparallel (S.R.), to a state in which both unpaired particles have spins parallel or antiparallel to their orbital angular momenta (W.R.). Here also there may or may not be a change of parity. Either or both unpaired particles may change parity.

(2) Going from a state in which both unpaired particles have spins parallel or antiparallel to their orbital angular momenta (W.R.) to another state in which also both unpaired particles have their spins parallel or antiparallel to their orbital angular momenta (W.R.). There is change of parity and only one of the unpaired particles changes in parity. Nb^{94} and Cs^{134} may be examples[3] of this case. If both unpaired particles change parity or not there is no change in parity.

These possibilities for the existence of long-lived odd–odd isomers explain the observations made by M. Goldhaber and A. W. Sunyar,[4] that odd–odd isomers appear to be unrelated to magic numbers and that they do not favor any particular parity change.

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