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NUCLEAR INSTRUMENTS AND METHODS 119 (1974) 295–299; © NORTH-HOLLAND PUBLISHING CO.

A SIMPLE TIME OPTIMAL DIGITAL CONTROL FOR NUCLEAR REACTORS

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Received 30 October 1973 and in revised form 7 March 1974

A time optimal control between two stationary power states of a nuclear reactor has been obtained with a digital computer using a simple algorithm, which provides a smooth approximation to the final state without overshoot. The results obtained by means of a digital simulation are given.

1. Introduction

Several authors have applied modern theories of optimal control to nuclear reactors, and particularly, have used the minimum time criterion. Among these authors there are some who have used Pontryagin's Maximum Principle, along which we intend to present our work.

The fundamental problem is the following: on the basis of the above mentioned principle, obtain a simple minimum time control for changing the power level in a nuclear reactor between two critical states. This is done by using a digital computer which takes sampled information from the reactor, processes it according to a program, and applies the results to the controlled process by means of an adequate signal.

The model of the reactor will be considered as[1,2])

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \lambda \rho n, \tag{1}$$

with

$$\lambda = 1/T, \quad T = l/k_{\mathrm{eff}}, \quad l = \sum \beta_i/\lambda_i,$$

where

T : period,

k_{eff} : $\dfrac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}}$,

β_i : fraction of delayed neutrons per group,

λ_i : decay constant of preceding group,

ρ : reactivity,

n : neutronic flux.

First, the optimum trajectory in the space state will be calculated for the desired power change and then, by means of a digital control based on the error between the measured and the required power, the system will tend to follow this trajectory.

At last, some experimental results obtained by means of a digital simulation of the system will be given.

2. Time optimal control

As we are interested in modifying the velocity of the reactivity variation in order to obtain an optimum behaviour, we analyze the following system[3]):

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \lambda \rho n, \qquad \frac{\mathrm{d}\rho}{\mathrm{d}t} = u. \tag{2}$$

Considering the characteristics of the reactor that will be controlled, we adopt the following limitations:

$$\rho_{\max} = 0.35\ \$\ ,$$

$$\left|\frac{\mathrm{d}\rho}{\mathrm{d}t}\right| \le u_0 = 0.0125\ \$/\mathrm{s}.$$

This means that the control region is closed, and that there is a restricted state variable.

The present analysis corresponds to an increase in power, though similar considerations can be made for a decrease.

The Hamiltonian for eqs (2) is

$$H = \psi_1 \lambda \rho n + \psi_2 u .$$

Calling u^* the time optimal control, and maximizing the expression above as a function of the variable u [Pontryagin's Principle[4])], we have

$$u^* = u_0 \operatorname{sg} \psi_2 ,$$

where $\operatorname{sg} x = 1$, if $x > 0$; $\operatorname{sg} x = 0$, if $x = 0$; $\operatorname{sg} x = -1$, if $x < 0$.

The adjoint variables verify the following equations:

$$\begin{aligned} \frac{\mathrm{d}\psi_1}{\mathrm{d}t} &= -\frac{\partial H}{\partial n} = -\psi_1 \lambda \rho, \\ \frac{\mathrm{d}\psi_2}{\mathrm{d}t} &= -\frac{\partial H}{\partial \rho} = -\psi_1 \lambda n, \end{aligned} \tag{3}$$

so that, from the first equations of eqs (2) and (3) we obtain

$$\psi_1 n = \text{constant} .$$

From these expressions we derive that $\psi_2(t)$ varies in a monotonous form, changing its sign only once at the most, and this is the reason why optimal control is $+u_0$ or $-u_0$ with only one change.

With the limitations imposed before, reactivity cannot exceed a prefixed value. Now we add the condition that the system must reach its final power without overshoot.

According to this, the useful region of the state space, as indicated in fig. 1, is limited by right lines $\rho = \rho_{max}$ and $n = n_r$.

It can be demonstrated that in this region the optimal trajectory will be the one indicated in fig. 1, which will be as follows: with maximum velocity until reaching the ρ_{max} boundary, evolution at constant reactivity $\rho = \rho_{max}$, and then with maximum negative velocity until reaching the final state. Segment AB (that lies on the boundary) and points A and B (where the trajectory enters and leaves the boundary) satisfy the conditions established in ref. 4 for the restricted state space, as can be seen following a similar process as that of refs 5 and 6.

3. Digital control

Using digital control we shall try to follow the optimal trajectory obtained in the last paragraph. This will be done analyzing each section of the curve.

Between points n_0 and A, the maximum velocity u_0 is used. Between A and B $\rho = \rho_{max}$ must be maintained; this is obtained making

$$\varepsilon = 1 - \rho/\rho_{max}, \tag{4}$$

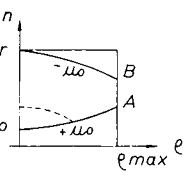

Fig. 1. Time optimal trajectory in restricted state space.

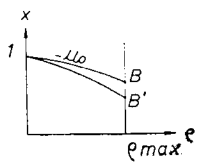

Fig. 2. Control change zone.

and as a first approximation we can consider control u as the following function

$$u = u_0 \operatorname{sg} \varepsilon . \tag{5}$$

From point B the control variable should take the value $-u_0$, but as the control is digital it is improbable that this will occur at exactly that point. So, to avoid inconveniences (overshoot, or that the system should end up in a different point to that which we desire), B was changed to a point B′ (fig. 2) such that when changing the control in B′, the most it will do is end up in B through $\rho = \rho_{max}$.

Clearly, from eq. (1) with $\lambda = 0.091$ [7], we see that calling $B = (\rho_0, x_1)$ and $B' = (\rho_0, x_1')$, we obtain

$$x_1' = x_1 \exp(-0.091\, \rho_{max} \Delta\tau),$$

where $x(t) = n(t)/n_r$ (normalized variable), and $\Delta\tau =$ = sampling period.

Giving ρ_{max} different values in the preceding expression we obtain a curve that, together with the curve of velocity $-u_0$, determines a region within which the program will order the change of u while the system evolves with $\rho = \rho_{max}$. From these conditions we obtain $x_1' = 0.60$.

If starting from B′ we use the maximum negative velocity to annul the reactivity, the system ends up at a lower power level than the expected one. But if simultaneously we use

$$\varepsilon = 1 - \frac{T_0}{T} + \frac{n}{n_r} \tag{6}$$

as an algorithm[8]) and maintain eq. (5) as control, the system tends to follow trajectories with $\varepsilon = 0$. These trajectories, which are right lines in the state space, intersect the axis at n_r.

It can be seen, that a control of the type given in eq. (5) produces limit cycles around the final state; this makes it necessary to introduce a proportional zone to avoid them (fig. 3).

Summarizing: in order to obtain our original objective with a digital control we use the expression (6)

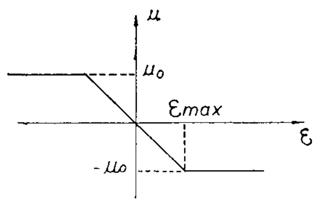

Fig. 3. Control based in the detected error.

for error and also a proportional zone for control indicated in fig. 3.

As expression (6) uses the variable T and as the accessible variable of the reactor is n we can calculate T as an approximation to the expression

$$T = n\left[\frac{\mathrm{d}n}{\mathrm{d}t}\right]^{-1}, \tag{7}$$

given by

$$T = \frac{\frac{1}{2}(n_4+n_0)}{\frac{1}{4}(n_4-n_0)/\Delta\tau} = 2\Delta\tau\,\frac{x_4+x_0}{x_4-x_0}, \tag{8}$$

where n_4 is the present value, and n_0 is the value of the sample taken four intervals before.

The control program in each sampling reads n, calculates T and determines the error. Consequently the simulated system diagram is the one shown in fig. 4.

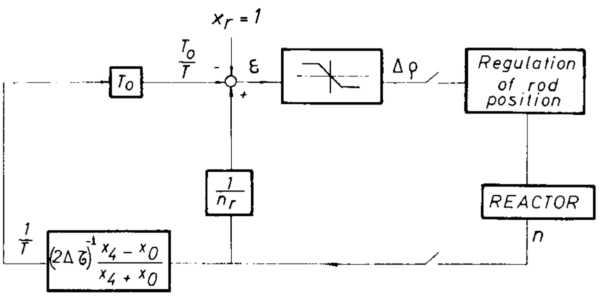

Fig. 4. Block diagram of simulated control system.

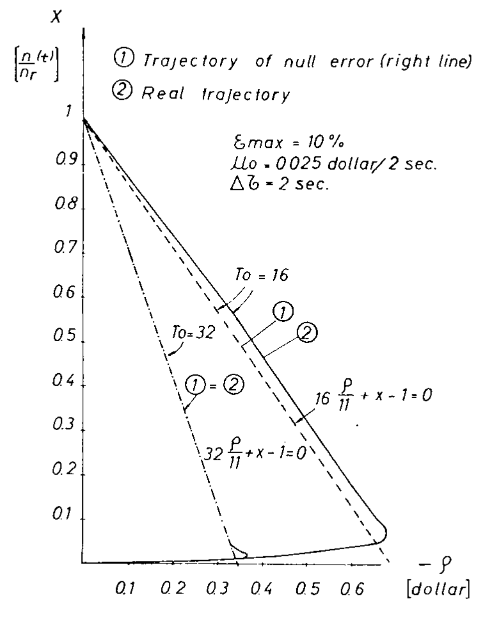

Fig. 5. Real trajectories and null-error right lines in state space for different T_0.

We will now analyze the behaviour of this system.

With model (1), expression (7) is reduced to

$$T = 1/\lambda\rho, \tag{9}$$

so that eq. (6) determines right lines in the state space, expressed by

$$\frac{1}{11}\,T_0\rho + x = 1, \tag{10}$$

using the same λ as before.

The system tends to follow these trajectories in the ideal case. In fig. 5 we have represented these right lines with zero error, and the real trajectories for different T_0 taking $\varepsilon_{max} = 10\%$ as an example.

It can be observed that the smaller the value of T_0, the greater the difference between the right line and the real trajectory. This is so, because the variation of x for the same power level in a given sampling interval is greater for larger values of ρ.

Several tests have been made with different values of ε_{max} between 20% and 1%, and it could be seen that for values of less than 8% the system started to oscillate when used with non-saturated pulses (fig. 6). Empirically it can be obtained that the period of the oscillations is constant ($8\Delta\tau$) and that the maximum amplitude is approximately $2u_0$ when the control is bang–bang.

In the next paragraph these oscillations will be analyzed and the gain determined.

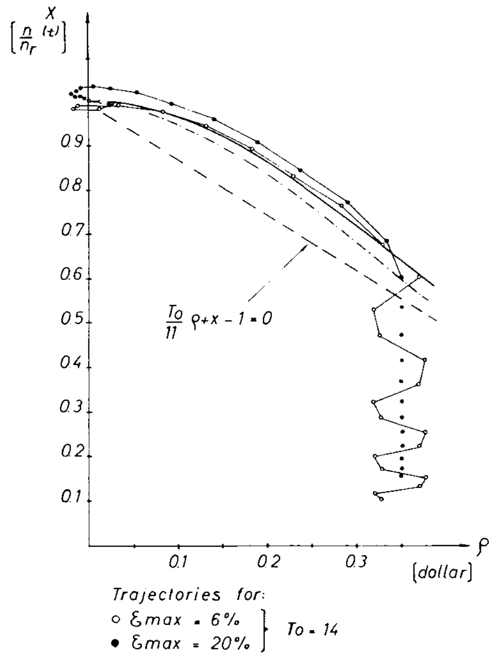

Fig. 6. Trajectories for different ε_{max}.

Evolution at constant reactivity is obtained modifying the control program according to eq. (4), so that the algorithm will be reduced to

$$\frac{T_0}{T} - 1 = \varepsilon . \tag{11}$$

At point B′ it must satisfy

$$\left| \frac{T_0}{T} + x_1' - 1 \right| \geq \varepsilon_{\max} , \tag{12}$$

with $T = 11/\rho_{\max}$ and $x_1' = 0.60$.

This indicates that T_0 and $\varepsilon_{\max}$ must be determined so that the system will not oscillate in the neighborhood of the final state.

4. Limit cycles

Experimentally it has been verified that with bang-bang control the system always produces a limit cycle of period $8\Delta\tau$ for any value of T_0 (fig. 7). The amplitudes of these cycles are approximately constant too for any value of T_0.

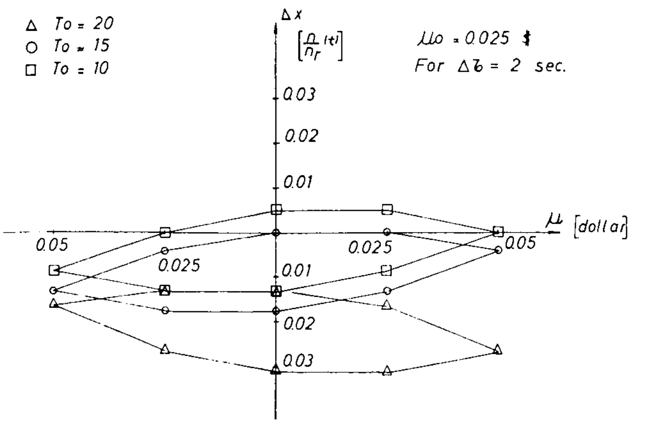

Fig. 7. Limit cycles in the neighborhood of the final state.

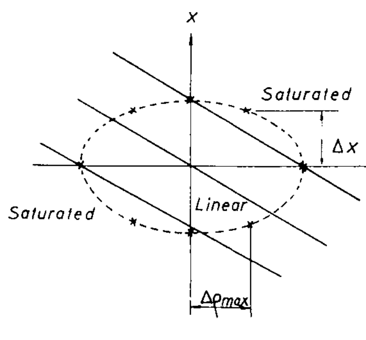

Fig. 8. Saturated and non-saturated zones of the control pulse.

These oscillations occur in zones where the system should maintain a constant reactivity: either when $\rho = \rho_{\max}$ (see section 3), or when $\rho = 0$ in the final state, i.e. when the control is not saturated.

If now we assume that the limit cycle is symmetric in respect to the right line parallel to axis ρ which passes through (0, 1), we obtain $\Delta x = 0.0183$ (fig. 8).

At this point we can determine the non-saturated control zone in the state space. As an approximation, the right lines parallel to the commutation one are

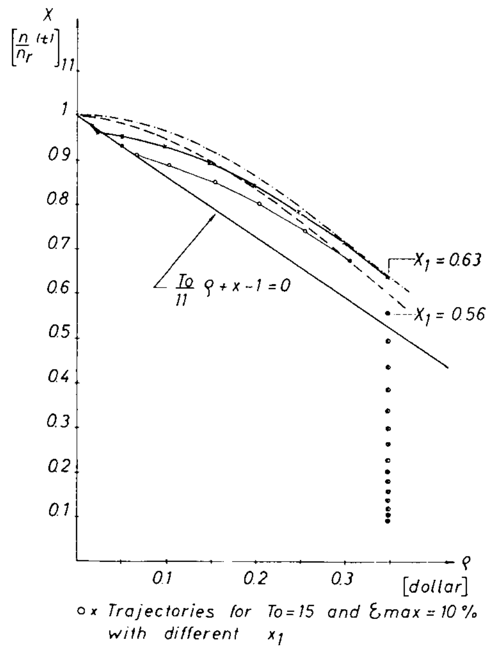

Fig. 9. Trajectories obtained using the control algorithm described.

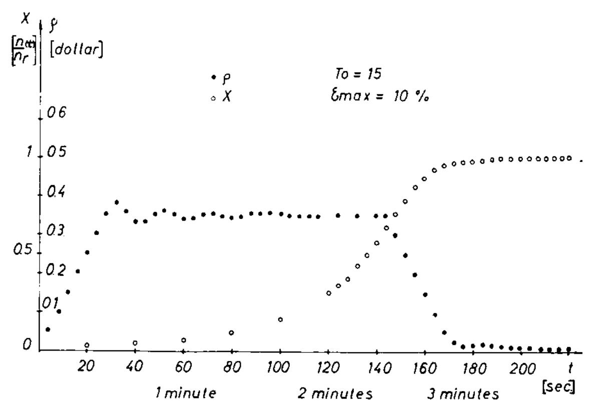

Fig. 10. Variations of normalized power and reactivity as a function of time.

drawn[9]). The equation for the commutation line is

$$\frac{T_0}{T}\,\rho + x = 1,$$

and the parallel ones which pass through $(\pm 2\Delta\rho_{\max}, 1)$ are

$$\frac{T_0}{T}\,\rho + x = 1 \pm \varepsilon_{\max}\,.$$

The following condition must also be verified:

$$\frac{1}{11}\,T_0\,2\Delta\rho_{\max} < \varepsilon_{\max}\,. \tag{13}$$

From eqs (12) and (13) we obtain $T_0 = 15$ and $\varepsilon_{\max} = 7\%$ as limit values.

With these values the oscillations on the $\rho = \rho_{\max}$ and at the desired equilibrium state disappear.

In fig. 9 the trajectory followed by the system for $\varepsilon = 10\%$ and $T_0 = 15$ can be observed. It can be seen that when the system leaves x_1', it develops a maximum-velocity trajectory (optimal), and in the neighborhood of the final state follows the corresponding right line, without oscillations or overshoot.

Fig. 10 shows the variations of normalized power and reactivity as a function of time.

References

1) L. E. Weaver, *Reactor dynamics and control* (American Elsevier, New York, 1968).
2) S. Fukutomi, private communication.
3) R. R. Mohler, Los Alamos Scientific Laboratory Report LA-3257-MS (1965).
4) L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and E. F. Mischenko, *The mathematical theory of optimal processes* (Wiley-Interscience Publ., New York, 1962).
5) K. Monta, J. Nucl. Sci. Techn. **3** (1966) 227.
6) K. Monta and S. Fukutomi, in Seminar on the *Application of on-line computers to nuclear reactors* (Sandefjord, 1968) paper OLC-8.
7) M. A. Schultz, *Control of nuclear reactors and power plants* (McGraw-Hill, New York, 1961).
8) Y. Takahashi, S. Takamatsu and K. Monta, Toshiba Rev. (Spring 1966) p. 5.
9) W. L. Nelson, Trans. ASME **83** (1961) 65.