Nilsson and Interacting-Boson-Model Pictures of Deformed Nuclei

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Variational solutions of the pairing-plus-quadrupole Hamiltonian are obtained for a system of protons and neutrons moving in a set of degenerate single-particle orbitals. The role played by pairs of particles coupled to angular momentum λ different from 0 and 2 has been studied. It is found that pairs of particles coupled to angular momentum $\lambda = 4$ are essential to obtain the properties of the ground state of deformed nuclei. Even pairs of particles coupled to angular momentum $\lambda = 6$ can play an important role in many cases.

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The two basic assumptions of the interacting-boson model (IBM) are that pairs of nucleons behave like bosons and that only $\lambda = 0, 2$ pairs are important for the description of quadrupole collective states (cf. Ref. 1 and references therein). In the present paper we study the validity of the second assumption for the case of strongly deformed nuclei. For this purpose we study the properties displayed by a system of protons and neutrons which move in a set of degenerate single-particle orbitals and interact through a pairing and a quadrupole residual force.

Two variational calculations are carried out based on the field approximation. The first corresponds to the quasiparticle Nilsson model,[2] where particles align in the average deformed field. Pairs of particles thus couple to all values of λ allowed by the angular momentum selection rules. In the second calculation pairs of particles are restricted to couple to $\lambda = 0$ and $\lambda = 2$. Although this last model is not the IBM it has much in common with it.

From the comparison of the results of the two models we try to answer to the question of whether the Hilbert space subtended by the pair-aligned model, which by construction is contained inside the particle-aligned model subspace, is large enough so that the properties of at least the ground state of a deformed nucleus are contained within it.

The variational wave function used in the calculations is written as

$$|\psi\rangle = \prod_{\nu, m>0} [U_\nu(m) + V_\nu(m) a^\dagger(\nu m) a^\dagger(\widetilde{\nu m})]|0\rangle . \quad (1)$$

The states $|\nu m\rangle$ are eigenstates of Y_{20} with eigenvalue $q_\nu(m)$. In the aligned fermion model the quantities $U_\nu(m)$ and $V_\nu(m)$ are the standard BCS occupation parameters, while in the pair-aligned model the occupation parameters are (cf. Bohr and Mottelson[3] and Broglia[3])

$$U_\nu(m) = \frac{1}{[1+\beta'^2 c_\nu^2(m)]^{1/2}} ,$$
$$V_\nu(m) = \frac{\beta' c_\nu(m)}{[1+\beta'^2 c_\nu^2(m)]^{1/2}} , \quad (2)$$

with

$$c_\nu(m) = \frac{\alpha_0}{\sqrt{\Omega}} + \frac{\alpha_2}{q} q_\nu(m), \quad (3)$$

where $q = [\sum_{\nu m} q_\nu^2(m)]^{1/2}$, $\sum_{\nu m} 1 = \Omega$ is the total number of pairs that the shells can accomodate, and β' is a normalization constant. The associated wave function $|\psi\rangle$ contains only pairs of particles coupled to angular momenta $\lambda = 0$ and $\lambda = 2$ with amplitudes α_0 and α_2, respectively, fulfilling the condition $\alpha_0^2 + \alpha_2^2 = 1$. The equilibrium parameters of the two models under discussion are obtained by solving the set of equations

$$\delta\{W - \lambda_N N\} = 0, \quad N = 2 \sum_{\nu, m>0} V_\nu^2(m), \quad (4)$$

where $W = \langle\psi|H|\psi\rangle = W_p + W_Q$ is the energy of the intrinsic state, the sum of the pairing (W_p) and the quadrupole (W_Q) energies; N is the average number of particles and λ_N a Lagrange multiplier.

(a) *Single-j shell.*—In this case we diagonalize a Hamiltonian consisting of a monopole pairing force with constant matrix elements G_0 acting between like particles and a quadrupole-quadrupole force of strength χ_{pn} acting between protons and neutrons. The ratio G_0/χ_{pn} between the coupling strengths is fixed to fulfill the condition that, for the number of pairs n leading to maximum deformation ($n/\Omega = 0.38$), the ratio $\Delta/\delta\epsilon$ between the pairing gap Δ and the total energy splitting $\delta\epsilon$ of the different magnetic substates of the j shell is of the order of 0.1. This condition makes our schematic model to resemble a "real nucleus" notwithstanding the obvious limitations.

The calculations were carried out for the case of a $j=\frac{41}{2}$ orbital which is approximately the degeneracy available to nucleons between shell closures 82 and 126. It was further assumed that the number of protons was equal to the number of neutrons. The predictions of the two models for different observables are shown in Fig. 1.

For the corresponding equilibrium parameters the associated energy is

$$W_p + W_Q = \begin{cases} -32.7 - 34.5 = -67.2 \text{ MeV } (SD) \\ -11.5 - 63.4 = -74.9 \text{ MeV } (\text{N+BCS}). \end{cases} \tag{5}$$

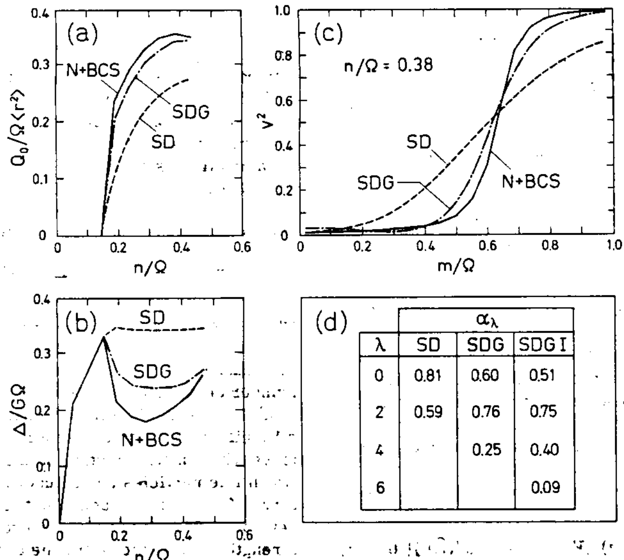
Thus, although the values of the equilibrium energy predicted by the two models differ by only 10%, the corresponding wave functions describe very different many-body systems. In fact $(\Delta)_{SD} \approx 1.7(\Delta)_{\text{N+BCS}}$ and $(Q_0)_{SD} \approx 0.7(Q_0)_{\text{N+BCS}}$, where Δ is the pairing gap and Q_0 the intrinsic quadrupole moment. One also finds that the average of the ratio $[V(\epsilon)_{SD}/V(\epsilon)_{\text{N+BCS}}]^2$ differs from 1 by ~45%, within the interval $\lambda_N - \frac{1}{2}\Delta \leq \epsilon \leq \lambda_N + \frac{1}{2}\Delta$ around the Fermi surface λ_N.

Writing the amplitude $c(m)$ in (3) as

$$c(m) = \Omega^{-1/2} \sum_\lambda (2\lambda+1)^{1/2} \langle jm\lambda 0 | jm \rangle \alpha_\lambda$$

with the condition $\sum_\lambda \alpha_\lambda^2 = 1$, one can extend the IBM to allow for any chosen set of values of[4] λ. This has been done for $\lambda = 0, 2, 4$ and $\lambda = 0, 2, 4, 6$. The results are also shown in Figs. 1(a)–1(c). Figure 1(d) shows the quantities α_λ for the three different pair-aligned model subspaces, i.e. (SD), (SDG), and $(SDGI)$. It is also noted that some aspects of the role played by pairs of particles coupled to angular momentum $\lambda = 4, 6, \ldots,$ have been studied in Ref. 5.

The most conspicuous feature of the results shown in Fig. 1 is that the inclusion of pairs of fermions with $\lambda > 2$, although they have small amplitudes,[6] increases the relative importance of the D pair with respect to the S pair. This is



λ	α_λ SD	α_λ SDG	α_λ SDGI
0	0.81	0.60	0.51
2	0.59	0.76	0.75
4		0.25	0.40
6			0.09

FIG. 1. Single j shell. (a) The intrinsic static quadrupole moment in untis of $\langle r^2\rangle\,\Omega$ as a function of the filling of the shell parameter n/Ω. The solid curve corresponds to the particle-aligned model (Nilsson plus BCS), while the dashed ($\lambda = 0, 2$) and dash-dotted ($\lambda = 0, 2, 4$) correspond to the predictions of the pair-aligned model (IBM) in the (S, D) and (S, D, G) subspaces, respectively. (b) The pairing gap parameter in units of $G\Omega$ as a function of n/Ω. (c) Occupation probabilities $V^2(m)$ for the models under discussion. (d) Amplitude that a pair of particles is in an angular momentum λ, when the minimization (4) is carried out in the (S, D), (S, D, G), and (S, D, G, I) subspaces.

because the presence of G and I pairs further breaks the superfluidity (monopole pairing correlations) and quadrupole-polarize the nucleus. With use of the SD part of the Nilsson-plus-BCS wave function ($\alpha_0 = 0.57$ and $\alpha_2 = 0.77$) to calculate the expectation value of the quadrupole moment, one obtains a result which is rather similar to that predicted by the Nilsson-plus-BCS model. Such calculations include renormalization effects arising from pairs of particles coupled to $\lambda = 4$ and 6, and have the "drawback" that they can be carried out only after the calculation in the full Nilsson-plus-BCS space has been done.

It is also pertinent to address to the question of the relevance of the overlaps between the SD wave function and the "exact" wave function. We have repeated the calculations whose predictions are shown in Fig. 1 for $j = \frac{15}{2}$. The corresponding results agree to the accuracy of the graphical display with the results associated with the $j = \frac{41}{2}$ orbital. However, $_{\text{N+BCS}}\langle\psi|\psi\rangle_{SD} = 0.53$ ($j = \frac{41}{2}$) and 0.76 ($j = \frac{15}{2}$).

(b) *Many-j shell.*—The Hamiltonian

$$H = V^{\text{SDI}}(p) + V^{\text{SDI}}(n) + V_{pn}, \tag{6}$$

where V^{SDI} indicates a surface δ interaction, was diagonalized in Ref. 6 in a set of degenerate single-particle orbitals composed by the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, and $2s_{1/2}$ orbitals and for $N = Z = 6$. In what follows we treat it in the Nilsson-plus-BCS approximation. The surface δ interaction can be approximated in terms of monopole and quadrupole pairing forces plus a quadrupole-quadrupole interaction. The relation between the coupling strengths of the last two components is fixed by demanding that for a system of identical particles, seniority is conserved and the energy of the lowest $v = 2$ state is independent of the number of particles. These are characteristic properties of the surface δ-interaction solutions. For a strength of V^{SDI} $g = -0.5$ MeV (cf. Ref. 7), the resulting value of the energy of the lowest 2^+ state calculated in the BCS plus random-phase approximation is 1.67 MeV. It agrees well with the value shown in Fig. 1 of Ref. 7.

The separate system of protons and neutrons is stable with respect to quadrupole deformations. It becomes deformed once the proton-neutron interaction is introduced. In this case, the quadrupole-quadrupole component in V^{SDI} also contributes to the splitting of the Nilsson levels. For a strength f of V_{pn} equal to -1.5 MeV (cf. Ref. 7) and for $N = Z = 6$ one obtains, solving Eq. (4), the results shown in Fig. 2. For this choice of the

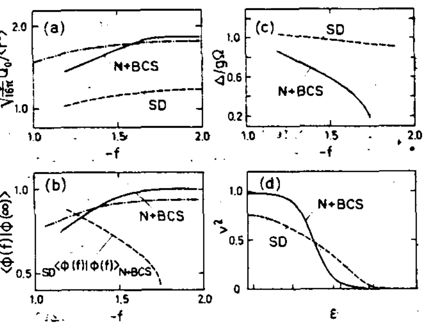


FIG. 2. Predictions associated with the degenerate many j-shell model of Ref. 7. (a) Intrinsic static quadrupole moment in units of $(16\pi/5)^{1/2}\langle r^2\rangle$, as a function of the coupling strength f. The dot-dashed curve corresponds to the results of Ref. 7. The solid curve corresponds to the diagonalization of essentially the same Hamiltonian, but in the quasiparticle approximation. The dashed curve indicates the predictions of the SD pair-aligned model (IBM). (b) Overlap $\langle\varphi(f)|\varphi(\infty)\rangle$ between the intrinsic state $\varphi(f)$ calculated for a given value of the strength parameter f and the intrinsic state calculated for $f = -\infty$ (totally aligned state). We also show the overlap $_{SD}\langle\varphi(f)|\varphi(f)\rangle_{\text{N+BCS}}$ between the Nilsson-plus-BCS and the SD intrinsic states, calculated as a function of f. (c) Pairing gap parameter in units of $g\Omega$ as a function of f. (d) Occupation probabilities $V_\nu^2(m)$.

parameters, $\Delta/\delta\epsilon \approx 0.13$. The system thus corresponds to a strongly deformed "nucleus."

While we make use of the BCS approximation to diagonalize the pairing interaction correlating the particles moving in the doubly degenerate magnetic substates, an exact diagonalization of V^{SDI} is carried out in Ref. 7. The observed discrepancies between the associated results are mainly due to particle number fluctuation, and are smaller than 10% [cf. Figs. 2(a) and 2(b)].

The present calculations fully confirm the main results obtained for a single-j shell. In particular $(\Delta)_{SD} = 1.7\,(\Delta)_{\text{N+BCS}}$, $(Q_0)_{SD} = 0.7\,(Q_0)_{\text{N+BCS}}$ and

$$W_p + W_Q = \begin{cases} -13.2 - 9.6 = -22.8 \text{ MeV } (SD) \\ -4.3 - 20.2 = -24.5 \text{ MeV(N+BCS)}. \end{cases} \tag{7}$$

It is noted that within the SD subspace Q_0 increases by only 15% in going from $f = -1.5$ to $f = -\infty$.

We conclude that the Nilsson-plus-BCS model and the IBM predict different results for the most important observables in strongly deformed nuclei. The most conspicuous differences appear in the gap parameter, in the quadrupole moment, and in the singie-particle occupation probabilities. The limitations implied by the *SD* subspace are thus not compatible with the coupling scheme appropriate for these nuclei. By extending the space to include pairs of particles coupled to 4 and 6, agreement is essentially obtained between the IBM and the Nilsson-plus-BCS model.

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[1]*Interacting Bosons in Nuclear Physics*, edited by F. Iachello (Plenum, New York, 1979).

[2]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Addison-Wesley, Reading, Mass., 1975), Vol. II.

[3]A. Bohr and B. R. Mottelson, Phys. Scr. 22, 468 (1980); R. A. Broglia, in *Interacting Bose-Fermi System in Nuclei*, edited by F. Iachello (Plenum, New York, 1981), p. 95.

[4]It can be shown that summing over all possible λ values one regains the Nilsson-plus-BCS model.

[5]J. Dukelsky, G. G. Dussel, and H. M. Sofia, Phys. Lett. 100B, 367 (1981); F. Iachello, in *Structure of Medium-Heavy Nuclei*, edited by the Demokritos Tandem Accelerator Group (The Institute of Physics, Bristol, 1980), p. 161; P. van Isacker, K. Heyde, M. Waroquier, and G. Wenes, Phys. Lett. 104B, 5 (1981); R. D. Ratna Raju, Phys. Rev. C 23, 518 (1981); K. A. Sage and B. R. Barrett, Phys. Rev. C 22, 1965 (1980); T. Otsuka, Nuci. Phys. A368, 244 (1981).

[6]In most of the discussions on the validity of the *SD* approximation which have been published to date, use is made of the *SD* part of the exact wave function rather than of the wave function obtained completely within the *SD* subspace [cf., e.g., T. Otsuka, Phys. Rev. Lett. 46, 710 (1981), and Nucl. Phys. A368, 244 (1981)].

[7]Otsuka, Ref. 6.