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Analysis of Inelastic Collision π^-+C at 915 MeV.

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Summary. — A calculation of some predictions of the independent particle model is described and compared with the experimental results on inelastic events occurring in the collision of π^--mesons on carbon nuclei, at the kinetic energy of 915 MeV. The experimental data are well described by the independent particle model when the inelastic pion-nucleon cases are assumed to be dominated by the $\frac{3}{2}$, $\frac{3}{2}$ resonance and the possibility of a second interaction of the pion within the nucleus is taken into account.

1. – Introduction.

There is a considerable amount of experimental information on the characteristics of pion-nuclei inelastic interactions up to energies of about 200 MeV (1-4). The characteristics of these interactions agree reasonably with the prediction of the independent particle model for the nucleus. The interaction is therefore described as taking place between the incident pion and one of the nucleons, as if both were free particles. The fact that the nucleons

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(1) G. BERNARDINI, E. T. BOOTH, L. LEDERMAN and J. TINLOT: *Phys. Rev.*, **80**, 924 (1950); **82**, 105 (1951).

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are actually bound is taken into account by assigning them momenta whose distribution, for the case of a carbon nucleus, can be well approximated by a gaussian distribution. Particles entering or leaving the nucleons will also be affected by the nuclear potential which keeps the nucleons « bound ».

It would be desirable to establish whether or not the independent particle model describes pion-nuclei interactions at higher energies as well.

We have already reported (5) the results obtained from interactions of negative pions of kinetic energy 915 MeV with carbon nuclei in a propane bubble chamber. An analysis of the diffraction scattering was performed, but the inelastic processes with re-emission of negative pions were just compared with hidrogen events at a similar energy without any interpretative discussion. The aim of this paper is to present a calculation of the predictions of the independent particle model, which are to be compared with the experimental results from the mentioned inelastic carbon events.

The calculations were performed in two steps: first we considered an interaction of the incoming pion with a single nucleon; this was treated with the help of the isobar model for pion production and using the Monte-Carlo method (Section **2**); secondly we estimated the effect of further interaction which may take place in the nucleus (Section **3**).

2. – The first interaction. Monte-Carlo calculation.

The basic assumption in the first step of our calculations will be the elementary interaction between negative pions of a fixed momentum and free nucleons whose momenta are distributed according to a gaussian. The effect of the nuclear potential which would produce a slight spread and variation in the energy of the incident pion beam, has been considered as negligible. Other corrections are discussed in Section **3**.

Since there is in ^{12}C equal number of protons and neutrons, the relative probability for a π^-+p or a π^-+n interaction has been directly taken as being proportional to the ratio between the total cross-sections for those two processes.

Once the pion-nucleon collision has taken place, an elastic or inelastic interaction can occur, with probabilities which will be energy-dependent. The total energy range was divided into a number of intervals depending on the available experimental information on the ratio between elastic and total cross-sections and on the differential cross-section for elastic events, which also varies with energy.

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(5) N. ABBATTISTA, M. BIASCO, S. MONGELLI, A. ROMANO, P. WALOSCHEK and E. PEREZ-FERREIRA: *Nuovo Cimento*, **23**, 1 (1962).

Three intervals were taken for the $\pi^- + p$ case, where experimental data on the ratio σ_{el}/σ_{tot}, $\sigma_{ch.ex.}/\sigma_{tot}$ and $d\sigma_{el}/d\Omega$ were available. These data were taken directly from experiments (6-9). For the $\pi^- + n$ case there is no direct experimental information, but assuming charge symmetry data on $\pi^+ + p$ collisions were taken as equivalent. Experiments on $\pi^+ + p$ collisions (10-11) are not so frequent as with $\pi^- + p$, and therefore, only two intervals could be taken for the $\pi^- + n$ cases.

Since the isobar model (12) has succeeded in describing sufficiently well, the pion production processes, in pion-nucleon interactions at energies within the range we are dealing with, it has been assumed that the inelastic processes take place through the excitation of the nucleon to the (3, 3) isobar state and the further decay of the isobar into a nucleon and a pion. Since in our kinematic calculations we will treat the isobar as a particle, we have not taken into account its very short life-time and, therefore, have considered it as having a unique mass which we took as 1230 MeV. The relative probabilities for the excitation of the different charge-states of the isobar, were calculated for the $\pi^- + p$ initial system on the basis of the values obtained by ALLES-BO-

TABLE I.

$$\pi^- + p \longrightarrow \begin{cases} \pi^- + p \\ \pi^0 + n \\ \pi^- + I^+ \longrightarrow \begin{cases} \pi^- + n + \pi^+ \\ \pi^- + p + \pi^0 \end{cases} \\ \pi^+ + I^- \longrightarrow \pi^+ + n + \pi^- \\ \pi^0 + I^0 \longrightarrow \begin{cases} \pi^0 + n + \pi^0 \\ \pi^0 + p + \pi^- \end{cases} \end{cases}$$

$$\pi^- + n \longrightarrow \begin{cases} \pi^- + n \\ \pi^0 + I^- \longrightarrow \pi^0 + n + \pi^- \\ \pi^- + I^0 \longrightarrow \begin{cases} \pi^- + n + \pi^0 \\ \pi^- + p + \pi^- \end{cases} \end{cases}$$

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RELLI *et al.* (^{12}) for the parameters of the isobar model. On the other hand since the $\pi^- + n$ system is a pure $T = \frac{3}{2}$ state, the relative probabilities for the different final states are given in a simple way by Clebsch-Gordan coefficients. Moreover, the angular distribution for the production of the (3, 3) isobar has been assumed to be isotropic, as well as its decay in its own c.m. frame of reference.

Consequently, all these cases summarized in Table I can occur, with relative probabilities which depend, as we have already said, on the total energy of the initial pion-nucleon system.

The different variables to which we have referred up to now, supply all the data one needs for the calculation of each simulated event. Their values will be fixed by random numbers. Therefore, 8 random numbers will be necessary for each case, giving the following data:

1) probability of collision with a proton or a neutron;

2) modulus of the momentum of the nucleon;

3) angle between the incident pion and the collided nucleon;

4) probability for having to deal with an elastic, charge exchange, or inelastic event, and in the latter case, with the excitation of a particular charged state of the isobar and its decay mode;

5) and 6) two angles giving the direction of the emerging particles in the c.m. system;

7) and 8) two angles giving the direction of decay products of the isobar in its own rest system.

Random numbers of two digits were used. The assignment of a particular value of momentum, angle, etc., has been done by dividing the corresponding probability function into 100 equally probable intervals and establishing a correspondence between the random numbers from 00 to 99 and the values of the variables in the mean point of each interval.

With those data one is able to evaluate the c.m.-energy of the pion-nucleon initial system and, therefore, the momentum of the particles A and B. If the case has turned out to be elastic, it will be enough to perform a Lorentz transformation into the laboratory frame of reference. For the particle A, representing the pion, both its momentum and angle with the incident pion should be transformed in order to get the two experimentally measured amplitudes. For particle B, which in the elastic case represents the nucleon, it will be enough to get its momentum in the lab. system in order to compare it with a maximum momentum value of the nucleon in C, so as to check if the case does not violate the exclusion principle. We accepted events only if the lab. momentum of the nucleon was greater than 200 MeV/c, corresponding to the maximum of a Fermi distribution. This is only a rough estimation of the Pauli principle effects, because we used a gaussian distribution for the momenta of the initial nucleons.

If, on the other hand, the case has turned out to be inelastic, one should point out firstly that the negative pion which is the only interesting particle from the experimental point of view, could be emitted either as particle A (the so-called extra-pion in the isobar model) or as particle D (decay-pion). In the first case, the momentum and angle of the π^- in the lab. system is obtained in an identical way as in the above described elastic case. For taking into account the exclusion principle, one should also evaluate the nucleon momentum in the lab. system. Since the nucleon in this case arises from the decay of the isobar, one must perform a double transformation of its momentum, first from the isobar c.m. system into the overall c.m. frame, and then from this into the lab. system.

Finally, in the cases in which the negative pion arises from the decay of the isobar, its momentum and angle with the incident pion direction must also pass through this double transformation.

The systematic calculations were performed by means of a Mercury computer, at the Computing Center of the Facultad de Ciencias at Buenos Aires.

3. – An estimate of the effect of a second interaction.

The model as described up to here presents many limitations. In fact, it neglects te potential effects, the many-body character of the problem, as well as the further interactions that may take place inside the nucleus.

At our energy (915 MeV) the real part of the nuclear potential is quite small as it was shown in the discussion of the diffraction scattering (⁵); its effects can certainly be neglected for the incoming pion and should not modify very much the outgoing one.

From another point of view it results impossible to consider in this model an exact treatment of the potential effects: as soon as particles interact inside the nucleus their potential energies change (potential energy depends on the type of the particles and on their energy). This change should naturally affect the kinematics of the reaction (particularly the momentum is not more conserved) making any further calculation cumbersome. This difficulty arises from the many-body character of the problem.

For these reasons we considered the possibility of a second interaction as the only correction worth taking into account. From the comparison of the calculated and the experimental momentum distribution of the re-emitted pions we could see that the main features of the experimental distribution were already reproduced by the one calculated in Section 2 but there were visible displacements of events towards the lower momenta, which should be due to the second interaction.

Of course, an accurate treatment of this second interaction would imply to repeat, for each re-emitted pion, the same overall process through which the incoming pion has passed. In turn, this would require careful considerations about the geometry of the nucleus and about cross-sections for pions of energies varying over a very wide range. This would enormously complicate our calculations in a way rather disproportional to the necessary approximations already introduced. We preferred, therefore, to make only a rough estimation of the effect of a second interaction, on the basis of very simple assumptions, namely the following:

i) That after the first interaction an average path in nuclear matter of about one C-radius is available for each emerging particle. The density of nucleons was assumed constant and ~ 0.052 nucleons/fermi^3 on this path.

ii) That mainly two types of interactions are important: for low energy pions the $\frac{3}{2}$, $\frac{3}{2}$ resonance (with some attenuation due to the Pauli principle) and for ~ 1 GeV pions an interaction very similar to the first one calculated by the Monte-Carlo.

Assumption ii) is justified by the particular grouping of our events: there are two well defined peaks in the spectrum obtained in Section 3: one around 300 MeV/c and the other at 1000 MeV/c.

The two main effects of the secondary interaction will be

a) The disappearance and regeneration of some π^- through the reactions

$$\pi^- + p \to \text{neutrals}\,,$$

$$\pi^0 + n \to \pi^- + p(+\pi^0)\,.$$

Therefore the number of events with 0, 1 or 2 negative prongs will be modified. The result of our calculation is shown in Table II.

b) The secondary interactions will also affect the momentum spectrum of the finally emitted pions.

TABLE II.

No. of negative prongs	Result of first interaction	After second interaction	Experimental data
0	403	627	701
1	2136	1868	1816
2	129	173	151
Total	2668	2668	2668

Low-energy pions will be scattered elastically by nucleons mainly in the $\frac{3}{2}$, $\frac{3}{2}$ state. Taking into accounto the Pauli principle which forbids forward collisions we were able to estimate the mean energy loss for these processes and their frequency.

High-energy pions will be elastically scattered by nucleons loosing a certain amount of energy. They will also produce additional pions. The result of the process can be evaluated from the Monte-Carlo data of the first interaction.

As a result of all these interactions the momentum spectrum is shifted towards lower momenta and the peak at ~1000 MeV/c is cut down. Figure 1 shows the momentum spectrum after the first interaction and the effect arising from the second one.

4. – Comparison with the experimental results.

Table II shows the number of events found with 0, 1 or 2 negative prongs. The Monte-Carlo calculation was made with a total of 2 668 events, the same

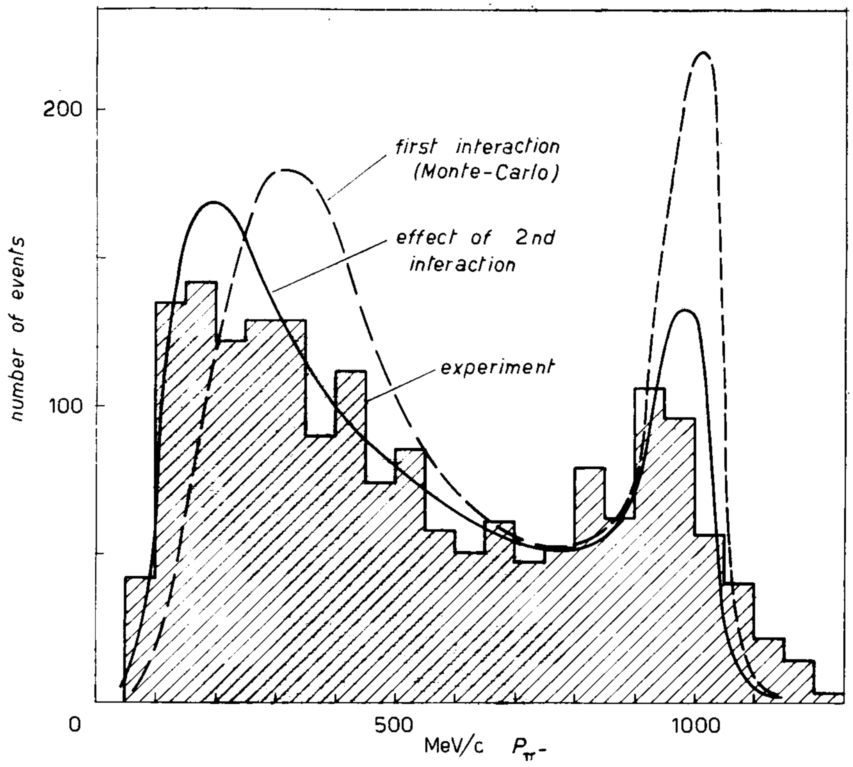

Fig. 1. – π^- momentum distribution.

number obtained in the experiment. Statistical errors can therefore be estimated directly from the given numbers. Figure 1 shows the momentum spectra obtained from the model calculation with a 2nd interaction. The experimental histogram was normalized to 1816 events (Table II) though only 1246 of them contained in a convenient fiducial region of the chamber, were actually measured.

In order to eliminate the diffraction scattering events from our experimental data we made a cut-off at 14° for events in which only a π^- is re-emitted at a momentum > 700 MeV/c. As it was shown in ref. (^{5}) the diffraction scattering is confined to this kinematical region. In our calculated spectrum we did the same cut-off to Monte-Carlo events in order to get comparable samples.

The final result of our calculation describes reasonably the experimental data. The independent particle processes assumed seem to justify the experimental facts and no more complicated absorption mechanism need to be taken into account. We may emphasize that the spectrum obtained from the model is not normalized to the experimental spectrum. Instead we just normalized the total number of events (total cross-section). For this reason the agreement in shape and area between experiment and model should be considered quite satisfactory.

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RIASSUNTO

In questo lavoro viene descritto un calcolo di alcune previsioni del modello a particelle indipendenti e paragonato ai risultati sperimentali sugli eventi anelastici nelle collisioni di mesoni π^- di 915 MeV di energia cinetica con nuclei di carbonio. I dati sperimentali sono spiegati bene da un modello a particelle indipendenti quando si fa l'ipotesi che gli eventi anelastici pione-nucleone siano dovuti essenzialmente alla risonanza $\frac{3}{2}$, $\frac{3}{2}$, e si tiene conto della possibilità di una seconda interazione del pione entro il nucleo.