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Quantitative High-Energy πp Scattering Predictions from Low-Energy Phase Shifts.

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Recently three detailed phase shift analyses of low-energy $\pi\mathcal{N}$ scattering have appeared, from groups at Berkeley (1), CERN (2) and Saclay (3). We apply continuous moment sum rules (4) to the above analyses in order to determine the relevant Regge contributions appearing in the $A'^{(-)}$, $B^{(-)}$, $A'^{(+)}$ and $B^{(+)}$ amplitudes (5).

A study of this kind, but with integer moment sum rules and the old Saclay phases, has been performed for $A'^{(-)}$ and $B^{(-)}$ by DOLEN *et al.* (6). Their conclusions were only qualitative; however, a well-established result of this work is the existence of a ϱ-zero in the $A'^{(-)}$ amplitude at $t \approx -0.2$ (which explains the cross-over phenomenon) and another ϱ-zero in the $B^{(-)}$ amplitude at $t \approx -0.6$ (which explains the dip in $\pi\mathcal{N}$ charge exchange (CEX) differential cross-section). BARGER and PHILLIPS (7) have also used continuous moment sum rules (with CERN phases) for $A'^{(+)}$ and $B^{(+)}$, in order to choose between different P, P' fits to the high-energy data. Their conclusions, however, are not satisfactory as far as $B^{(+)}$ is concerned.

Our considerations are strictly quantitative and their aim is *to predict*, with a reasonable error, the high-energy $\pi\mathcal{N}$ scattering data near the forward direction ($t \geqslant -0.4$) from the low-energy phases. The predictions we get are in good agreement with the ex-

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(1) C. H. JOHNSON: UCRL preprint 17683 (1967).

(2) A. DONNACHIE, R. G. KIRSOPP and C. LOVELACE: CERN preprint TH. 838, Addendum (1967).

(3) P. BAREYRE, C. BRICMAN and G. VILLET: *Phys. Rev.*, **165**, 1730 (1968).

(4) M. G. OLSSON: *Phys. Lett.*, **26** B, 310 (1968); A. DELLA SELVA, L. MASPERI and R. ODORICO: *Nuovo Cimento*, **54** A, 979 (1968).

(5) We use the notation of V. SINGH: *Phys. Rev.*, **129**, 1889 (1963), and G. F. CHEW, M. L. GOLDBERGER, F. E. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1337 (1957).

(6) R. DOLEN, D. HORN and C. SCHMID: *Phys. Rev. Lett.*, **19**, 402 (1967); *Phys. Rev.*, **166**, 1768 (1968).

(7) V. BARGER and R. J. N. PHILLIPS: *Phys. Lett.*, **26** B, 730 (1968).

isting high-energy experimental data. In $A'^{(-)}$ we find evidence (from the CERN phases) for a Regge cut which gives the correct $\pi\mathcal{N}$ CEX polarization. In $B^{(+)}$ a Regge singularity with $\alpha \simeq -0.5, -1$ is found which contributes negligibly to $A'^{(+)}$. This new singularity can explain the energy behaviour of $P(\pi^+p) + P(\pi^-p)$.

For the $A'^{(-)}$ amplitude the following sum rule appears:

$$(1)\qquad \varphi(\gamma) = \nu_{\max}^{-(\gamma+1)}\left\{\frac{8\pi^2 f^2}{\mu^2}\,\frac{t/2-\mu^2}{1-t/4M^2}\cdot|\nu_B^2-\nu_0^2|^{\gamma/2} + \int_{\nu_0}^{\nu_{\max}}|\nu^2-\nu_0^2|^{\gamma/2}\,\mathrm{Im}\left[\exp\left[-i\frac{\pi}{2}\gamma\right]A'^{(-)}(\nu,t)\right]d\nu\right\} = \sum_k \frac{\beta_k}{\cos(\pi/2)\alpha_k}\,\frac{\sin(\pi/2)(\alpha_k+\gamma+1)}{\alpha_k+\gamma+1}\,\nu_{\max}^{\alpha_k},$$

where, at high energy,

$$(2)\qquad A'^{(-)} \simeq \sum_k \beta_k(t)\,\frac{1-\exp[-i\pi\alpha_k]}{\sin\pi\alpha_k}\,\nu^{\alpha_k}.$$

Similar form have the sum rules for $\nu B^{(-)}$, $\nu A'^{(+)}$ and $B^{(+)}$.

Results.

1) $A'^{(-)}$. The three phase-shift analyses considered agree in predicting (*) the ϱ to vanish at $t \approx -0.2$ and to be negligible thereafter, thus suggesting that it also vanishes at $\alpha = 0$. But there is no agreement about what remains after the ϱ disappears. Saclay and Berkeley produce a term with $\alpha \approx -1$, independent of t, which can be interpreted as a Regge background contribution. However, the CEX polarization predicted ($\sim 8\%$ at 5.9 GeV, $\sim(3\div4)\%$ at 11.2 GeV) is too small and falls down too rapidly with the energy. Instead, the CERN analysis gives very particular $\varphi(\gamma)$'s of the kind shown in Fig. 1. It is not possible to give a natural explanation of these curves in terms of few Regge poles and background. They may be fitted only by a continuous distribution in the J-plane, *i.e.* by a Regge cut. We have considered a family of cuts with discontinuities of exponential shape, parametrized as follows:

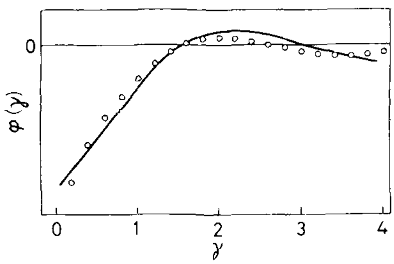

Fig. 1. – Comparison of $\varphi(\gamma)$ (see eq. (1)) for $A'^{(-)}$ at $t = -0.2$ (isolated points) with a pure exponential cut contribution (solid line) having $\tau = 2.3$ (see eq. (3)).

$$(3)\qquad A'^{(-)}_{\text{cut}}(\nu,t) = i\exp\left[-i\frac{\pi}{2}\alpha_c\right]\beta_c\nu^{\alpha_c}\frac{1}{\tau}\int_{-\infty}^{\alpha_c}d\alpha\exp\left[\frac{\alpha-\alpha_c}{\tau}\right]\exp\left[-i\frac{\pi}{2}(\alpha-\alpha_c)\right]\nu^{\alpha-\alpha_c} = i\exp\left[-i\frac{\pi}{2}\alpha_c\right]\beta_c\cdot\nu^{\alpha_c}\frac{1}{\tau[\ln(\nu\exp[1/\tau])-i\pi/2]}.$$

(*) Only the CERN analysis covers the complete interval from the maximum energy considered (1.935 GeV in this case) down to threshold. The Berkeley analysis covers the interval (490÷1566) MeV and the Saclay analysis the interval (310÷1560) MeV (laboratory kinetic energy of the pion). In order to use the Berkeley and Saclay phases in our sum rules, we have filled the gap from threshold to the minimum energy considered with the CERN phases.

For the $\varphi(\gamma)$'s under consideration the fit gives (assuming $\alpha_c = 0.58$) a $\tau \approx 2.3$ in the range $-0.4 \leqslant t \leqslant -0.2$. The β_c turns out to vary slowly with t and to have a sign opposite to the ϱ residue.

In a previous analysis (see DELLA SELVA *et al.* ([4])) of forward $\pi\mathcal{N}$ CEX scattering using total cross-section data, a second contribution with $\alpha \approx -1$ was found besides the ϱ. This does not contradict CERN $\varphi(\gamma)$'s, because the system ϱ + cut, when the ϱ is dominant, admits a good parametrization in terms of the ϱ and a second pole with a lower α. Also the $\varphi(\gamma)$ determined from the CERN analysis allows at $t = 0$ a really good fit of this kind, giving $\alpha_\varrho(0) = 0.58$ and $\alpha = -1$ for the second pole.

Because of the zero at $t \approx -0.2$ and the presence of the cut, one cannot expect a good determination of the ϱ slope from $A'^{(-)}$. This parameter has been determined from the $B^{(-)}$.

2) $B^{(-)}$. All three low-energy analyses give essentially the same curves for this amplitude. From them it is clear that the ϱ is dominant and vanishes at $t \approx -0.55$; however it is also clear that something else ($\sim 10\%$ at 1.9 GeV) is present. We have tried several kinds of fit (ϱ + background, $\varrho + \varrho'$, ϱ + cut) to establish the nature of this second contribution, but no conclusive answer was found. Anyway, the sign of this contribution is such that the polarization (essentially deriving from interference of ϱ in $B^{(-)}$ and cut in $A'^{(-)}$) is depressed for $t \geqslant -0.15$, and therefore it has a negligible weight in the CEX polarization because of the zero of the ϱ in $A'^{(-)}$.

Unfortunately, the presence of this second Regge singularity does not allow a good determination of the ϱ trajectory from a direct inspection of the curves at each value of t. We note indeed that the determination of α is very sensitive to the presence of small contributions beside the dominant one. We have preferred to use the fact that the t behaviour of the ϱ residue in $B^{(-)}$, as determined from the sum rules, strongly suggests for it a form $\sim \alpha_\varrho(t) \cdot \exp[bt]$. Assuming this form and $\alpha_\varrho(0) = 0.58$ one then finds $\alpha'_\varrho(0) = 0.96\ (\text{GeV})^{-2}$ and $b = 1.75\ (\text{GeV})^{-2}$. The residue has been determined from the maxima of the $\varphi(\gamma)$, interpreting them as pure ϱ maxima; the error committed is likely to be no more than $(3 \div 4)\%$.

In conclusion, we have obtained the following parametrization for the $A'^{(-)}$ and $B^{(-)}$ amplitudes:

$$(4)\quad \left\{\begin{aligned}
&\alpha_\varrho(t) = 0.58 + 0.96\,t\,,\\
&\tau = 2.3 && \text{in } (\text{GeV})^2\,,\\
&A'^{(-)}(\nu, t) = \alpha_\varrho(32\cdot\exp[2.65\,t] - 12)\,i\exp\left[-i\frac{\pi}{2}\alpha_\varrho\right]\nu^{\alpha_\varrho} -\\
&\qquad -(10/\tau)\,i\exp\left[-i\frac{\pi}{2}\,\alpha_\varrho(0)\right]\left[\ln\left(\exp[1/\tau]\nu/1.9\right) - i\frac{\pi}{2}\right]^{-1}\nu^{\alpha_\varrho(0)} && \text{in } (\text{GeV})^{-1}\,,\\
&B^{(-)}(\nu, t) = \alpha_\varrho\cdot 185\exp[1.75\,t]\,i\exp[-i(\pi/2)\,\alpha_\varrho]\,\nu^{\alpha_\varrho - 1} && \text{in } (\text{GeV})^{-2}\,.
\end{aligned}\right.$$

This parametrization is valid only up to $-t \sim 0.4 \div 0.5$. Thereafter many symptoms indicate that the truncated Legendre series becomes a bad approximation to the amplitudes and the investigation cannot proceed further.

In Fig. 2 the predictions for the differential cross-section at 5.9 and 18.2 GeV and for the polarization at 5.9 and 11.2 GeV are compared with the data of Stirling *et al.* [8] and Bonamy *et al.* [9], respectively. The predicted polarization curve shown in the Figure is truly depressed for $t \geqslant -0.15$ by the not-well-determined secondary contribution in $B^{(-)}$; this effect increases approaching $t = 0$.

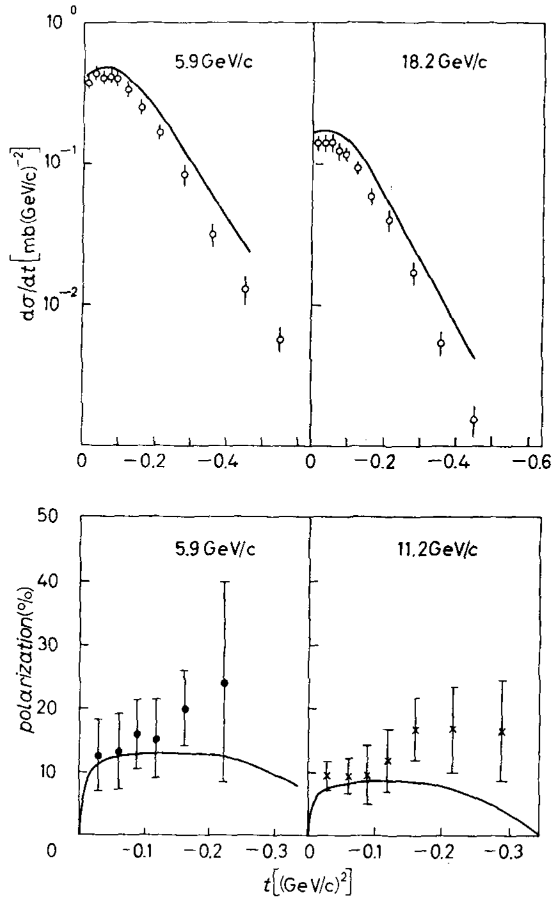

Fig. 2. – Comparison of $\pi^- p \to \pi^0 n$ differential cross-section and polarization calculated from eq. (4) with a sample of experimental data [8,9].

3) $A'^{(+)}, B^{(+)}$. For $A'^{(+)}$ and $B^{(+)}$ there is also good agreement between the $\varphi(\gamma)$ derived from Saclay, Berkeley and CERN analyses.

Barger and Phillips [7] have already considered these sum rules and, comparing them with high-energy fits, have concluded that they are well explained by the P, P' system of Regge poles. We should, however, point out a difficulty one meets in drawing this conclusion. If one makes a two-pole fit to $B^{(+)}$ at $t = 0$, taking $\alpha = 1$ for one of

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([8]) A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzasio and B. Amblard: *Phys. Rev. Lett.*, **14**, 763 (1965).

([9]) P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. P. Guillaud, J. Schneider, M. Yvert, I. Manelli, F. Sergiampietri and L. Vincelli: *Phys. Lett.*, **23**, 501 (1966); and *Proc. of the Heidelberg Int. Conf. on Elementary Particles* (1967).

them, $\alpha < 0$ is obtained for the other pole. This cannot reasonably be imputed to errors, because the agreement between the different phase-shift analyses is, in this case, excellent. To re-establish agreement with the P, P' picture one has to introduce a third pole with $\alpha \sim -0.5, -1$ contributing negligibly to $A'^{(+)}$.

Instead of enlarging the already large P, P' set of parameters, we have preferred to parametrize $A'^{(+)}$ and $B^{(+)}$ by two Regge cuts with exponential-shape discontinuities and with branch points fixed at $\alpha_c = 1$. This means two parameters to be extracted from each curve at each value of t considered, and prediction of the high-energy differential cross-sections and polarization data becomes possible.

The best fits to the $\varphi(\gamma)$ curves of $A'^{(+)}$ and $B^{(+)}$ are excellent up to $-t \leqslant 0.3$; thereafter discrepancies begin to appear. The analysis gives

$$(5)\quad \begin{cases} A'^{(+)} = i\,99 \exp[3.0\,t] \dfrac{\nu}{\tau_A[\ln((\nu/1.9)\exp[i/\tau_A]) - i\pi/2]} & (\text{GeV})^{-1}, \\ \tau_A = 0.18 - 0.68\,t & (\text{GeV})^2, \\ B^{(+)} = i\,143 \exp[1.64\,t] \dfrac{1}{\tau_B[\ln((\nu/1.9)\exp[1/\tau_B]) - i\pi/2]} & (\text{GeV})^{-1}, \\ \tau_B = 0.77 - 1.05\,t & (\text{GeV})^2. \end{cases}$$

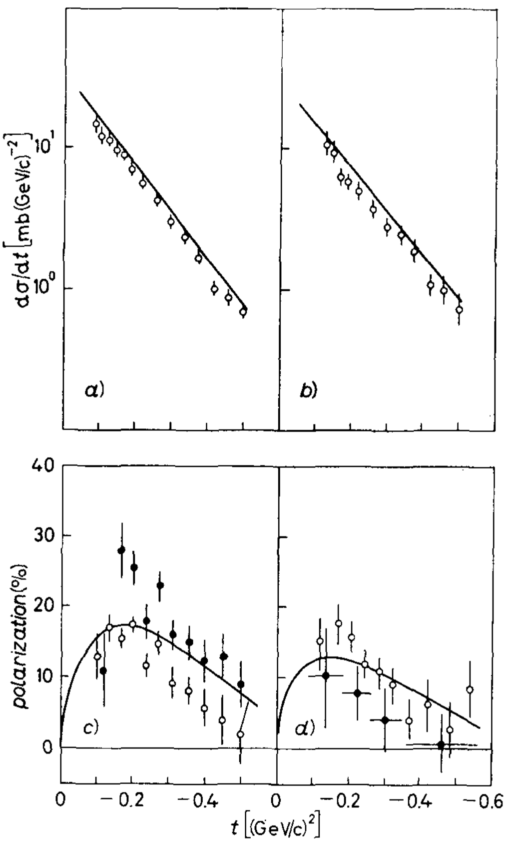

Fig. 3. – In the upper figure: comparison of π^-p and π^+p elastic cross-sections calculated from eq. (5) with the data of HARTING *et al.* [10] at 12.4 GeV; *a*) π^-p; *b*) π^+p. In the lower figure: $\frac{1}{2}[P(\pi^+p) - P(\pi^-p)]$ calculated from eq. (5) (solid curves) is plotted together with the experimental data [11] for $P(\pi^+p)$ and $-P(\pi^-p)$ at 6 and 12 GeV/c: *c*) 6 GeV/c; *d*) 12 GeV/c; o π^-p; • π^+p.

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[10] D. HARTING, P. BLACKALL, B. ELSNER, A. C. HELMHOLZ, W. C. MIDDELKOOP, B. POWELL, B. ZACHAROV, P. ZANELLA, P. DALPIAZ, M. N. FOCACCI, S. FOCARDI, G. GIACOMELLI, L. MONARI, J. A. BEANEY, R. A. DONALD, P. MASON, L. W. JONES and D. O. CALDWELL: *Nuovo Cimento*, **38**, 60 (1965).

[11] M. BORGHINI, G. COIGNET, L. DICK, K. KURODA, L. DI LELLA, P. C. MACQ, A. MICHALOWICZ and J. C. OLIVIER: *Phys. Lett.*, **24** B, 77 (1967).

The predictions for the high-energy differential cross-sections of π^-p and π^+p are in good agreement with the data of Harting *et al.* [10] (see Fig. 3). In particular the cross-over effect is correctly reproduced. The prediction for the difference $\frac{1}{2}[P(\pi^+p) - P(\pi^-p)]$ ($\sim \operatorname{Im}(A'^{(+)}B^{(-)*})$) of π^+p and π^-p polarizations is also satisfactory (see Fig. 3). As to $\frac{1}{2}[P(\pi^+p) + P(\pi^-p)]$ ($\sim \operatorname{Im}(A'^{(+)}B^{(+)*})$), it is predicted positive in agreement with the data of Esterling *et al.* [12] at 5.15 GeV and the data of Borghini *et al.* [11] at 6 and 10 GeV [13] but too large in magnitude (at 10 GeV one gets $\sim +20\%$ against $(+2 \pm 5)\%$). This means that the approximation of assuming the discontinuities of the cuts to have a pure exponential form is too crude. To restore agreement between the sum rules and the polarization data it is sufficient to suppose that in $B^{(+)}$, beside the exponential cut, some further singularity (a pole or a bump in the cut discontinuity) is present near $\alpha \approx -0.5, -1$ with a « residue » having the same sign of the exponential cut discontinuity and a weight at 2 GeV of the order of 10% of the exponential cut weight.

Conclusion. Starting from the low-energy (below 2 GeV) phase-shift analyses of Berkeley, CERN and Saclay, and using continuous moment sum rules, the high-energy differential cross-sections and polarizations of πp CEX and $\pi^\pm$p elastic scattering have been predicted up to momentum transfers $-t \leqslant 0.3 \div 0.4$. The analysis of the CERN phases (which are in contradiction on this point with those of Berkeley and Saclay) gives the result that in the $A'^{(-)}$ amplitude besides the ϱ a cut is present, which is put in evidence by the ϱ dynamical zero at $t \approx -0.2$. This cut correctly predicts the $\pi\mathcal{N}$ CEX polarization. Some weak evidence for the ϱ choosing nonsense at $\alpha = 0$ is also provided. The analysis was not able to establish the nature of the secondary contribution ($\sim 10\%$ of the ϱ at 2 GeV) found to be present in $B^{(-)}$.

As to $A'^{(+)}$ and $B^{(+)}$ the comparison of the data for $P(\pi^+p) + P(\pi^-p)$ with the sum rules produces evidence for the existence in $B^{(+)}$ of a Regge singularity with $\alpha \sim \sim -0.5, -1$, which contributes negligibly to $A'^{(+)}$. This Regge singularity can explain the features of the high-energy data for $P(\pi^+p) + P(\pi^-p)$ observed at present.

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[12] R. J. Esterling, N. E. Booth, G. Conforto, J. Parry, J. Scheid and D. Sherden: paper submitted to the XIV Int. Conf. on High-Energy Physics (Vienna, 1968).

[13] The negative value of $P(\pi^+p) + P(\pi^-p)$ at 12 GeV which results from the data of Borghini *et al.* [11] is not confirmed by the more recent data of Bell *et al.* [14] on π^+p polarization at 14 GeV.

[14] R. T. Bell, M. Borghini, L. Dick, G. Gregoire, L. di Lella, J. C. Olivier, M. Poulet, P. Scharff-Hansen, D. Cronenberg, K. Kuroda, A. Michalowicz, G. Bellettini, P. L. Braccini, T. Del Prete, L. Foà, G. Sanguinetti and M. Valdata: paper presented at XIV Int. Conf. on High-Energy Physics (Vienna, 1968), and private communication from Drs. G. Bellettini and L. di Lella.