HIGH-LYING PAIRING RESONANCES★

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Pairing vibrations based on the excitation of pairs of particles and holes across major shells are predicted at an excitation energy of about $70/A^{1/3}$ MeV and carrying a cross section which is 20%–100% the ground state cross section.

The existence of a pairing condensate characterizes the low energy excitation spectrum of nuclei far away from closed shells. This feature is specifically demonstrated by the concentration of $L = 0$ cross section associated with two nucleon transfer reactions. In fact, the ground state is essentially the only $J^{\pi} = 0^{+}$ state[‡1] populated in such processes (cf. e.g. ref. [1]).

The modes generated by fluctuations of the pairing field [4, 5] play also an important role in the case in which particles and holes can be clearly distinguished. Thus e.g. in closed shell nuclei [4, 6], where the average value of the pairing gap is zero. It is the purpose of present note to show that the role of the pairing vibrations in the nuclear spectrum is ubiquous, as the distinction between particles and holes can always be achieved in terms of excitations across major shells[‡2].

We discuss first the case of normal systems. The corresponding low-lying pair addition and pair subtraction modes have been extensively studied. They display a cross section roughly proportional to the pair degeneracy ($\frac{2}{3}A^{2/3}$) of the corresponding harmonic oscillator valence shell. Note that it is also possible to create pairing phonons in higher and lower shells. Therefore, states with an energy of the order of $2\hbar\omega$ and with a cross section similar to the ground state cross section are expected to be excited in two-nucleon transfer processes.

With the random phase approximation (RPA) the energy of the lowest giant pairing mode is given by the dispersion relation

$$\frac{1}{G} = F(W) = \frac{4\hbar\omega\Omega}{(2\hbar\omega)^2 - W^2}. \quad (1)$$

In the above estimate a pairing force with constant matrix elements G was utilized. The value of this constant to be used depends on the number of levels in which the interacting particles are allowed to move. It has been empirically determined that, for the case of three harmonic oscillator shells, $G \approx (17/A)$ MeV. Thus,

$$E \approx 0.84\,(2\hbar\omega) \approx (68/A^{1/3})\ \text{MeV}. \quad (2)$$

The cross section associated with this mode is proportional to

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[‡1] Departures from this systematics can be traced back to shell effects (see e.g. refs. [1–3]).

[‡2] Little is known on the distribution of pair transfer strength but the for low energy part of the nuclear spectra. This distribution should be strongly affected by the existence of major nuclear shells such that the inter-shell distance is appreciably larger than the distance between the levels within a shell. In such cases a concentration of pairing strength in a single state is expected for each major shell. In the present note we confine the discussion to the lowest of the high lying pairing resonances.

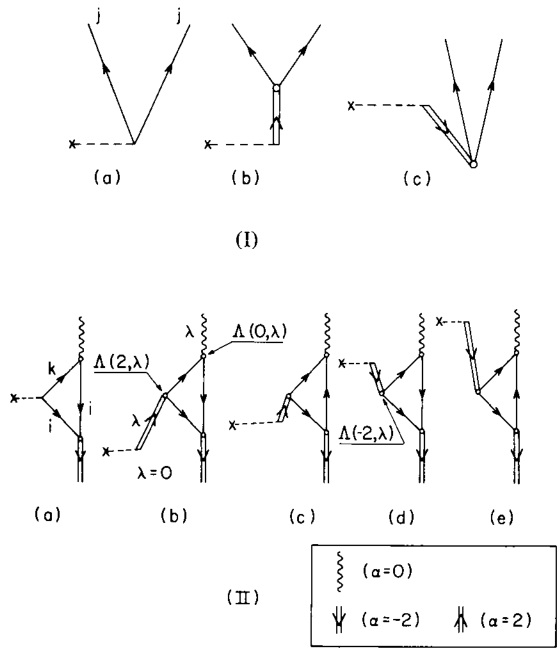

Fig. 1. Effective two-nucleon transfer amplitude. The cross followed by the dotted line represents the two-nucleon transfer external field. The strength with which the different modes couple with the fermion field is $\Lambda(\alpha, \lambda)$. (I) Bare and lowest order renormalization contribution to the transfer amplitude associated with the excitation of a pure configuration $j^2(0)$. The single arrowed line indicates a particle while the double arrowed lines represents the giant pairing mode. The contribution of graph (c) is due to the presence of ground state correlations. A typical example of the process illustrated by graph (a)–(c) is found in the excitation of the different two-particle configuration of the ground state of ^{210}Pb in the reaction ^{208}Pb(t, p)^{210}Pb(gs). (II) Bare and lowest order renormalization contribution to the excitation of a particle-hole mode in closed shell nuclei (wavy line). The two-nucleon transfer field (cross followed by dotted line) creates either a particle k and destroys a hole i or creates the giant pair addition or giant pair removal mode. The initial state ($\alpha = -2$, $\lambda = 0$) is the ground state of the nucleus with two nucleons removed from closed shell. A typical example of the process illustrated by graphs (a)–(e) is found in ^{206}Pb(t, p)^{208}Pb (3^-; 2.62 MeV).

$$\langle \text{giant} | T(\lambda = 0) | 0 \rangle^2 = (\Lambda/G)^2 \approx 1.2\ \Omega, \tag{3}$$

where the two-body transfer operator is defined as

$$T(\lambda = 0) = \sum_j B(j, j; \lambda = 0)\, \frac{[a_j^+ a_j^+]_{00}}{\sqrt{2}}, \tag{4}$$

with

$$B(j, j; \lambda = 0) = \sqrt{\frac{2j+1}{2}} = \sqrt{\Omega}. \tag{5}$$

The B-coefficients are the two-body spectroscopic amplitudes associated with the two-particle configuration $j^2(0)$. The quantity Λ is determined by the equation

$$\Lambda = \left[\frac{\mathrm{d}F(W)}{\mathrm{d}W}\right]^{-1/2} = \sqrt{\frac{2\hbar\omega}{W}}\, G\Omega^{1/2}. \tag{6}$$

The giant pairing mode couples to pairs of particles and holes with a strength Λ. Such coupling renormalizes the two-nucleon transfer cross sections associated with the low-lying pairing modes. In fig. 1(I) are displayed the lowest order contributions to the renormalization of the B-coefficients calculated in the framework of the nuclear field theory (NFT) [7]. We thus obtain

$$B(jj; \lambda = 0) = B(jj; \lambda = 0)_{\text{bare}} + B(jj; \lambda = 0)_{\text{pol}} \tag{7}$$

where the first term is the bare amplitude, while

$$\frac{B(jj; \lambda = 0)_{\text{pol}}}{B(jj; \lambda = 0)_{\text{bare}}} = \frac{\Lambda}{G}\left(\frac{1}{2\epsilon_j - W} - \frac{1}{2\epsilon_j + W}\right) \approx \frac{2\Lambda^2}{GW} = \left(\frac{2\hbar\omega}{W}\right)^2 \frac{G\Omega}{\hbar\omega} \approx 0.41. \tag{8}$$

Here ϵ_j is the energy of the single-particle orbital to which the two nucleons are transferred. In deriving (8) it was assumed that $\epsilon_j \gg W$. The cross section associated with the low-lying pair addition and pair removal modes, are thus renormalized by a factor $(1 + B_{\text{pol}}/B_{\text{bare}})^2 \approx 2$, because of the presence of the giant pairing modes. Note that the effective two-nucleon spectroscopic amplitude plays a similar role in two-nucleon transfer reactions than the one played by the effective charge in electromagnetic processes. The above discussion is illustrated by the results displayed in table 1. They were obtained diagonalizing the pairing force in a three level model consisting of two shells above the Fermi surface, and one shell below it.

The fact that a significant part of the transition to the lowest states takes place via the giant pairing mode (renormalization effect) strongly reduces the difference between the cross section associated with pure two-particle configurations[‡3]. A typical example

[‡3] For a discussion of the role of the "hot orbitals" in two-nucleon transfer reactions cf. e.g. [8].

Table 1
Excitation energy and two-particle transfer cross section of the giant pairing modes. In all cases a three level model was utilized, each level having a pair degeneracy equal to Ω. The distance between the levels is $\hbar$ and the Fermi surface was set half way between the first and second level. The results labeled (a) correspond to the solutions obtained allowing only excitations from the lowest to the middle level (low-lying) and from the lowest to the highest level (giant). In Column (b) we display the results of allowing simultaneous excitations between the first and second and first and third level, i.e. the full solution of the problem. In all cases the RPA was utilized.

	$(E/\hbar\omega)$		(α/Ω)	
	(a)	(b)	(a)	(b)
Low-lying	0	0	1.6	2.2
Giant	2	2.2	1.2	0.8

of this effect is provided by the estimate of the ^{208}Pb(t, p)^{210}Pb (L = 0) cross sections, in which the particles are allowed to move only in the valence shell. Excited states with energies around 3.5 MeV are predicted, displaying cross sections which are 1–3 times larger than the cross sections associated with the ground state transitions[‡4]. The expected intensities are reduced to ~1/3 of the ground state intensity when the particles are allowed to correlate in a large number of shells, i.e., when the renormalization effects of the high lying pairing modes are taken into account (for more detail cf. ref. [9], in particular table 3). Experimentally no $J^\pi = 0^+$ has been observed except for the ground state.

The situation discussed above is very reminiscent of e.g. the response of the nucleus to an external quadrupole electric field which changes in time. In fact, in the case of Coulomb excitation there are "hot" orbitals (protons) as well as "cold" orbitals (neutrons). The polarization charge induced by the giant quadrupole resonances tends to equalize the effective charges associated with both kind of particles. This is why the main Coulomb excitation strength is found, in the low energy spectra and usually in a single quadrupole state in even-even nuclei.

The distinction between particles and holes is also achieved in superfluid nuclei for the case of two-particle, two-hole (2p–2h) excitations across a major shell, i.e. excitations of energy $2\hbar\omega$.

‡4 This result can be understood in terms of the large cross section associated with the $d^2_{5/2}(0)$, $s^2_{1/2}(0)$ and $d^2_{3/2}(0)$ configurations (cf. table 2d, ref. [8]).

The main properties of the giant pairing modes in nuclei far away from closed shells can be obtained utilizing again the three level model discussed above. In this case however, the lowest level is completely filled while the middle level is half filled. The corresponding BCS gap equation reads

$$1 = \frac{G\Omega}{E} + \frac{G\Omega}{2\Delta}, \tag{9}$$

where

$$E = (\hbar\omega)^2 + \Delta^2, \tag{10}$$

is the quasiparticle energy. Eq. (9) leads to

$$\Delta \approx \frac{7.8}{A^{1/3}} \text{ MeV}. \tag{11}$$

The excitation energy of the giant pairing vibration is fixed by the dispersion relation

$$\frac{1}{G} = F(W) = \frac{(2\hbar\omega)^2}{E(4E^2 - W^2)}. \tag{12}$$

We thus obtain

$$W = \left[\frac{G\Omega}{2\Delta} + \left(\frac{\Delta}{\hbar\omega}\right)^2\right]^{1/2} \approx 1.8\,\hbar\omega \approx \frac{72}{A^{1/3}} \text{ MeV}, \tag{13}$$

and

$$\sigma \propto (\Lambda/G)^2 \approx 0.6\,\Omega. \tag{14}$$

The giant pairing mode thus implies the coherent excitation of a number $\approx\sqrt{\Omega}$ of 2p–2h configurations of energy $2\hbar\omega$. Note that (13) and (14) coincide, within 20%, with the corresponding schematic model values obtained from nuclei around closed shell (cf. columns b of table 1). Thus, the properties of the giant pairing mode are expected to be rather independent of the particular valence shell of the nucleus under consideration.

The previous estimate of the excitation energy corresponds to the rather extreme situation in which all levels belonging to a major shell are degenerate. If this is not the case, the energy of the pairing resonance should decrease as the lowest shell is being fulfilled. The energy variation of the mode with mass number is expected to be about twice the change in the energy of the Fermi surface in the same mass span, e.g. about 3 MeV between ^{114}Sn to ^{124}Sn. This quantity provides with a rough estimate of the uncertainty associated with the estimate (13). On the other hand, (14) should be rather independent of the change in the Fermi energy. The expected behaviour of the giant pairing modes is quite different from the behavior of the properties characterizing the ground state, i.e., the lowest pair addition or pair removal modes. In particular the ground state cross section‡5 changes from $\sigma \propto (\Delta/G)^2 \approx 0.5\ \Omega^2$ (superfluid systems) to $\sigma \propto 2.2\ \Omega$ (normal systems). Note also that in superfluid nuclei, the population of the giant pairing mode is estimated to be considerably weaker than the population of the ground state‡6.

The above discussed properties of the giant pairing modes correspond quite closely to the known properties of the giant quadrupole resonances (cf. ref. [5]). In fact the B(E2)-value associated with these resonances change very smoothly with mass number and for deformed nuclei is much smaller than the B(E2)-value associated with the ground state rotational band 2^+ state. While the number of two-fold degenerate levels participating in the giant quadrupole resonance is $\approx\sqrt{\Omega}$, those participating in the 2_1^+ is $\approx \Omega$.

There is, however, an important difference between the surface modes and the pairing modes. The gap in the 2p–2h excitation spectrum is decreased by the presence of surface modes and it eventually disappears for surface deformed nuclei. In such case, the concentration of two-particle transfer strength in a single state cannot be expected. On the other hand, there is always a gap in the particle-hole excitation spectrum of a given spin and parity. This is because major nuclear shells alternate in parity. Thus, giant surface vibrational modes are also to be found in shape deformed nuclei and they are expected to be, in general, more stable than the giant pairing modes as a function of mass number.

The giant pairing mode will appear, for nuclei far away from closed shell, in a region where the density of levels is large and will thus display a finite width. Around closed shells, part of the strength of the giant resonance is concentrated in the lowest $2m$ (particles) $-2n$ (holes) modes, examples of which are the 4.8 MeV and 5.3 MeV states in ^{208}Pb and ^{206}Pb, respectively, and the 0^+ states around 5.5 MeV observed in the Ca-isotopes (cf. e.g. ref. [1]).

The generalization of the concept of giant pairing modes to multipolarities other than $\lambda = 0$ can be carried along the same lines discussed above utilizing a multipole pairing interaction [3, 11]. The corresponding dispersion relation can also be cast in the form (1). Defining the effective λ-pole pair degeneracy as

$$\Omega(\lambda) = \frac{4\pi}{2\lambda + 1} \sum_{j,j'} |\langle j || Y_\lambda || j'\rangle|^2 \tag{15}$$

the values of $G(\lambda)$ to be used are equal for the different multipolarities and coincide with $G = G(\lambda = 0)$ [12]. Note that $\Omega(\lambda = 0) = \Sigma_j \sqrt{j + \frac{1}{2}}$.

The generalized two-particle transfer operator is

$$T(\lambda,\mu) = \frac{4\pi}{2\lambda+1} \sum_{j \geqslant j'} \frac{\langle j || Y_\lambda || j'\rangle}{\sqrt{1+\delta(j,j')}} \frac{[a_j^+ a_{j'}^+]_{\lambda\mu}}{\sqrt{1+\delta(j,j')}} . \tag{16}$$

The associated cross section is again proportional to $(\Lambda(\lambda)/G(\lambda))^2$ (cf. eq. (3)), where $\Lambda(\lambda)$ is defined in (6). For quadrupole modes

$$\Omega(2) \approx \tfrac{5}{16} N^2 \approx \tfrac{5}{12} A^{2/3} \approx 0.62\ \Omega \quad (\lambda = 0). \tag{17}$$

Thus

$$W(2)/W(0) = 1.1, \tag{18}$$

and

$$\sigma(2)/\sigma(0) = (\Lambda(2)/\Lambda(0))^2 \approx 0.8. \tag{19}$$

We now discuss the renormalization effects associated with the population, in two-particle transfer processes, of a $\Delta N = 0$ particle-hole mode in closed shell nuclei. In fig. 1 (II) are displayed the bare and lowest order polarization diagrams in the language of the NFT. Labeling the different phonons by (α, λ), α being the

‡5 The renormalization of this cross section induced by the $2\hbar\omega$ pairing modes is reflected in the strength of the effective pairing coupling constant needed to reproduce a given value of the pairing distortion. In fact, the value of G to be used in the three level model is about 30% smaller than the corresponding value needed in the one-level model, if the magnitude Δ is to be kept fixed.

‡6 For $A \sim 100$ the predicted cross section associated with the giant pairing mode is ~10% of the ground state cross section. The result of a detailed calculation may be rather different from this model result, although not in its order of magnitude. Thus, an analysis of the low-lying pairing modes excited in the ^{116}Sn(t, p)^{118}Sn reaction predicts states at about 9 MeV of excitation energy ($2\hbar\omega$ type of states) carrying a summed relative cross section ~20% of the ground state cross section (cf. table 10 of ref. [10]). Note, however, that this number can become as large as ~30% for triton energies such that the mismatch between entrance and exit channel is minimized (cf. fig. 9, ref. [10]).

transfer quantum number (i.e. $\alpha = 0$ for particle-hole modes and ± 2 for pairing modes) we obtain (cf. (7))

$$\frac{B(\mathrm{ki};\lambda=2)_{\mathrm{pol}}}{B(\mathrm{ki};\lambda=2)_{\mathrm{bare}}} = \frac{\Lambda^2(2\lambda)}{G[W_1(-2,0)+W(2,\lambda)-W(0,\lambda)]} + \frac{\Lambda^2(-2,\lambda)}{G[W(0,\lambda)+W(-2,\lambda)-W_1(-2,0)]} \approx \frac{2\Lambda^2(2,\lambda)}{GW(2,\lambda)}, \tag{20}$$

where the assumption has been made that $\Lambda(2,\lambda) \approx \Lambda(-2,\lambda)$, $W(2,\lambda) \approx W(-2,\lambda)$ and $W(2,\lambda) \gg |W(0,\lambda) - W_1(-2,0)|$, and where the subindex 1 labels the lowest pairing mode.

For $\lambda = 2$ we obtain that (20) is ≈ 0.21. The two-nucleon transfer cross section associated with the $(\alpha, \lambda) = (0, 2)$, $\Delta N = 0$ mode, is thus increased by about 1.5 because of the presence of the quadrupole pairing resonance.

Note that this estimate refers to the contribution of the high lying pairing mode to the two-particle spectroscopic amplitude of the surface vibration. The approximations leading to (20) cannot be used in estimating the corresponding contribution of the low-lying multipole pairing modes. In fact, these low-lying modes do not give rise to a renormalization effect, but their interplay with the particle hole mode has to be worked out explicitly in the different cases.

We conclude that the source of renormalization effects for two-particle transfer processes is to be found in the giant pairing resonances in a similar way as the giant particle-hole modes are the source of effective charges, effective magnetic moments, etc. The lowest pairing modes are expected at an energy of about $70/A^{1/3}$ MeV and carrying a cross section which is equal to ~0.2–1.0 times the cross section of the lowest pair addition or pair removal mode.

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References

[1] R.A. Broglia, O. Hansen and C. Riedel, in: Adv. in Nucl. Phys., vol. 6, eds. M. Baranger and E. Vogt (Plenum Press, New York 1973) p. 287.
[2] R.A. Broglia, D. Kunz and B.S. Nilsson, Phys. Lett. 38B (1972) 383.
[3] I. Ragnarsson and R.A. Broglia, Nucl. Phys. A263 (1976) 315.
[4] D. Bes and R.A. Broglia, Nucl. Phys. 80 (1965).
[5] A. Bohr and B. Mottelson, Nuclear structure, vol. II (Benjamin, New York, 1975).
[6] E. Flynn, G. Igo and R.A. Broglia, Phys. Lett. 41B (1972) 397.
[7] D.R. Bes et al., Phys. Lett. 52B (1974) 253; R.A. Broglia et al., Phys. Lett. 64B (1976) 29; D.R. Bes et al., Nucl. Phys. A260 (1976) 77.
[8] R.A. Broglia, C. Riedel and T. Udagawa, Nucl. Phys. A184 (1972) 23.
[9] E.R. Flynn et al., Nucl. Phys. A195 (1972) 97.
[10] R.A. Broglia, C. Riedel, B. Sørensen and T. Udagawa, Nucl. Phys. A115 (1968) 273.
[11] D.R. Bes, R.A. Broglia and B. Nilsson, Phys. Lett. 40B (1972) 338.
[12] R.A. Broglia, D.R. Bes and B.S. Nilsson, Phys. Lett. 50B (1974) 213.