Quantum corrections to the Vlasov description of collective modes

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Abstract

Quantum corrections to the Vlasov approach using an initial Thomas-Fermi distribution are estimated by comparing with exact transition densities for quadrupole and octupole collective excitations in a schematic model. We show that corrections depend both on the dynamics and the initial state, and found them to be relevant for the low lying states.

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In te last years the Vlasov approach to nuclear dynamics became a well established framework to describe collective excitation as well as fragmentation processes arising in Heavy Ion collisions [1]. It provides an economic alternative to a full quantum calculation in the nuclear many-body problem. Moreover, two-body effects can be taken into account through the incorporation of a collision term, extending the original mean field description [2].

Recently Brink et al. [3] have developed a Vlasov approach to study small amplitude nuclear collective excitations. Further modifications have been introduced [4] to somehow include spin and quantum effects in this otherwise classical linear response theory. The scheme has been particularly fruitful in the qualitative and quantitative description of the strength for collective modes at low and intermediate energies for both isoscalar [5] and isovector [6] excitations. The approach has also been exploited in atomic and solid state physics [7].

It has been argued that if the initial phase space distribution fulfills the Pauli exclusion principle, the classical evolution implicit in the Vlasov equation, does not violate this quantum condition. The consideration of fermion statistics, introduced as an initial condition, has lead to think of the Vlasov approach as of semiclassical character.

The need of quantum corrections to the nuclear Vlasov equation has been discussed by several authors [4,8,9]. Most of the previous work within the Vlasov approach to collective excitations has focused on the weaker test of predicting the energy spectra, for which the Vlasov results are quite satisfactory [3-4]. Quantum corrections to transition densities and other fluid-dynamical quantities have been considered [8-9] in a much more stringent test of the theory. However, to perform the analysis [8-9], as well as in

most of the applications of the Vlasov approach [3-6], it has always been assumed that the initial configuration of the nuclear system can be represented by a Thomas-Fermi (TF) structureless distribution instead of the exact one.

Quantum corrections to the Vlasov approach involve consideration of the terms neglected in the classical evolution and the use of the exact initial distribution instead of semiclassical approximations to it. The aim of the present work is to point out that care should be taken when trying to evaluate these quantum corrections, since they are highly dependent on the particular initial state considered. This last point is certainly relevant at zero temperature. To show this we solve a schematic model [8,10-11], at both the quantum and classical level. We then evaluate the quantum corrections to the Vlasov approach by considering the exact quantal Wigner distribution, or alternatively the TF distribution as the initial condition.

Let us consider A nucleons in a spherical ground state $\hat{\rho}_0$, of a 3D harmonic oscillator mean field $\hat{H}_0 = H[\hat{\rho}_0] = \hat{p}^2/2M + M\omega_0^2\hat{r}^2/2$. We will study the excitation of the system under the action of $\hat{V}_{ext} = \beta(t)\,\hat{Q} = \beta(t)\, r^\lambda\, Y_{\lambda 0}(r)$, a local external field of definite multipolarity λ.

The departure from the equilibrium initial configuration is given by $\delta\hat{\rho}(t) = \hat{\rho}(t) - \hat{\rho}_0$, while $\hat{W} = H[\hat{\rho}(t)] - \hat{H}o$ is the dynamic self-consistent contribution to the Hamiltonian. Assuming a separable two-body residual interaction of multipole-multipole type and neglecting exchange, we have

$$\hat{W}(\hat{r}, t) = k_\lambda\, \hat{Q}(\hat{r})\, \langle Q\rangle(t) \qquad (1)$$

where $\langle Q\rangle(t) = \mathrm{Tr}[\delta\hat{\rho}(t)\, \hat{Q}(\hat{r})]$ and the coupling k_λ is obtained from the self-consistency condition. This model has been recurrent in the description of isoscalar excitations both in microscopic quantum RPA

calculations [10] and within the Vlasov approach [8,11].

As we are interested in describing small amplitude motion, the dynamical evolution of the system will be governed by

$$\left[\frac{\partial}{\partial t} - Ho\ \Lambda \right]\delta f = \left[\frac{2}{\hbar}\, Q(\mathbf{r})\, \sin(\frac{\hbar}{2}\, \Lambda)\, f_0(\mathbf{r},\mathbf{p}) \right] \left[\beta(t)+k_\lambda <Q>(t)\right] \quad (2)$$

where $H_0(\mathbf{r},\mathbf{p})$, $Q(\mathbf{r})$ and $f(\mathbf{r},\mathbf{p},t) = f_0(\mathbf{r},\mathbf{p}) + \delta f(\mathbf{r},\mathbf{p},t)$ are the Weyl-Wigner phase space functions associated with the one-body operators $\hat{H}_0$, $\hat{Q}$ and $\hat{\rho}/(2\pi\hbar)^3$ respectively (the last is traced on spin and isospin indexes). The differential operator Λ in Eq.(2) is the one associated to the Poisson bracket, i.e. $A(\mathbf{r},\mathbf{p})\ \Lambda\ B(\mathbf{r},\mathbf{p}) = \partial A/\partial \mathbf{r} \cdot \partial B/\partial \mathbf{p} - \partial A/\partial \mathbf{p} \cdot \partial B/\partial \mathbf{r}$.

The inhomogeneous Liouville like Eq.(2) can be solved by the Green function technique with the propagator given by $\delta[\mathbf{r}'-\mathbf{r}_0(\mathbf{r},\mathbf{p},t)]\ \delta[\mathbf{p}'-\mathbf{p}_0(\mathbf{r},\mathbf{p},t)]$, i.e., the Dirac delta function in the classical unperturbed (backward in time) trajectories [8,11]: $\mathbf{r}_0(\mathbf{r},\mathbf{p},t)$ and $\mathbf{p}_0(\mathbf{r},\mathbf{p},t)$ are the initial phase space coordinates of a particle that reach $(\mathbf{r},\mathbf{p})$ at time t. For the Fourier components we get $\delta f(\mathbf{r},\mathbf{p},\omega) = \chi^0(\mathbf{r},\mathbf{p},\omega)\ \beta(\omega) + [k_\lambda\ <Q>(\omega)]$ where χ^0 denotes the undressed *phase space* susceptibility:

$$\chi^0(\mathbf{r},\mathbf{p},\omega) = \mathcal{F}\left\{ \frac{2}{\hbar}\, Q(\mathbf{r})\, \sin(\frac{\hbar}{2}\, \Lambda)\, f_0(\mathbf{r},\mathbf{p}) \right]_{\mathbf{r}_0,\mathbf{p}_0},\ \omega \Big\}. \quad (3)$$

From this we can evaluate the transition density $\delta n(\mathbf{r},\omega) = \mathrm{Tr}[\delta\hat{\rho}\ \delta(\hat{\mathbf{r}}-\mathbf{r})] = \int d\mathbf{p}\ \delta f(\mathbf{r},\mathbf{p},\omega)$, among other fluid-dynamical quantities. The response, $\mathbb{R}_Q(\omega) \equiv \beta^{-1}(\omega)\ <Q>(\omega)$ and the strength function, $S_Q(\omega) \equiv -\pi^{-1}\ \mathcal{I}m\ [\ \mathbb{R}_Q(\omega)\ ]$, can also be readily obtained. The only effect of the separable interaction is to shift the energy of the modes and redistribute their strength according to $\mathbb{R}_Q(\omega) = \mathbb{R}^0_Q(\omega)\ /\ [1-k_\lambda\ \mathbb{R}^0_Q(\omega)]$ where the superscript denotes undressed quantities.

It follows from Eq.(3) that the difference between the Vlasov and

the exact quantum results is due to neglecting the higher-orders terms in $\hbar$ in the dynamics, and replacing of the exact initial ground state distribution f_0 by the semiclassical TF distribution. Certainly, Eq.(3) is the Wigner phase space representation of the Fourier transform of the commutator $(i\hbar)^{-1}\,[V^H_{ext}(t'),\ \rho^H_0(t)]$ (the operators are in the Heisenberg picture), i.e., the generalized susceptibility as derived in the linear response theory of a many-body system. To lowest order in $\hbar$, Eq.(3) becomes

$$\chi^0(\mathbf{r},\mathbf{p},\omega) \propto \mathcal{F}\left\{ \frac{\partial Q}{\partial \mathbf{r}}\frac{\partial f_0}{\partial \mathbf{p}}\right]_{\mathbf{r}_0,\mathbf{p}_0},\ \omega \right\} \equiv \chi^0_{cl}(\mathbf{r},\mathbf{p},\omega) \qquad (4)$$

i.e., the *classical* susceptibility. In the last equation, to be consistent with the approximations made, $f_0(\mathbf{r},\mathbf{p})$ must be replaced by $f_0^{TF}(\mathbf{r},\mathbf{p})$. However, to evaluate the quantum corrections to the Vlasov approach, the true determinantal ground state should be considered instead. Higher order terms in $\hbar$, left out in Eq.(4) are:

$$\chi^0_{qc}(\mathbf{r},\mathbf{p},\omega) = \mathcal{F}\left\{ -\frac{\hbar^2}{24}\sum \frac{\partial^3 Q}{\partial r_i \partial r_j \partial r_k}\,\frac{\partial^3 f_0}{\partial p_i \partial p_j \partial p_k} + O(\hbar^4)\ \right]_{\mathbf{r}_0,\mathbf{p}_0},\ \omega \right\} \qquad (5)$$

The first multipolarity for which dynamic quantum corrections contribute is the octupole (λ=3) one. For spin and isospin saturated *magic nuclei* with $A=2Z=2N=\frac{2}{3}[(K+2)^3-(K+2)]$ and $K=0,1,\ldots$ being the last occupied major oscillator shell, the exact ground state Wigner distribution is analytically known [12]. We can then compare, for the fluid dynamical quantities, the exact results with those obtained with the usually adopted TF distribution with and without the dynamic quantum corrections. In this letter we concentrate the analysis on the transition density for both quadrupole (λ=2) and octupole (λ=3) excitations.

For λ=2, quantum corrections arise from shell effects absent in the TF distribution. In the present model only one collective (giant

resonance) state exists, and thus the spatial structure of the transition density, consistent with the doorway hypothesis, is $\delta n(r,\omega) \propto r\, dn_0/dr$. The quantum oscillations due to the use of the exact initial distribution are not important. The Vlasov-Thomas-Fermi description is reasonably good and also the energy of the mode is very well predicted.

According to Eqs. (3-5) we derive for the λ=3 excitation

$$\delta n^0(r,\omega) = \beta(\omega)\, Y_{30}(r)\Big\{ \Big[\delta n_{cl}(r) + 3\,\delta n_{qc}(r)\Big]\, I^-(\Omega^0_{LEOR},\omega) + \Big[\delta n_{cl}(r) - \delta n_{qc}(r)\Big]\, I^-(\Omega^0_{HEOR},\omega)\Big\}, \qquad (6)$$

where we have introduced $I^{\pm}(\Omega,\omega) = (\hbar\omega-\hbar\Omega+i\eta)^{-1} \pm (\hbar\omega+\hbar\Omega+i\eta)^{-1}$, the low (LEOR) and high energy (HEOR) modes without interactions: $\hbar\Omega^0_{LEOR} = \hbar\omega_0$, $\hbar\Omega^0_{HEOR} = 3\hbar\omega_0$. The classical and quantum contributions to the transition density are

$$\delta n_{cl}(r) = \frac{3}{8}\frac{\hbar}{M\omega_0}\, r^2\, \frac{dn_0}{dr}$$
$$\delta n_{qc}(r) = -\frac{1}{32}\left(\frac{\hbar}{M\omega_0}\right)^3 \left\{ \frac{d^3}{dr^3} - \frac{3}{r}\frac{d^2}{dr^2} + \frac{3}{r^2}\frac{d}{dr} \right\} n_0(r) \qquad (7)$$

It is evident from Eq. (6) that quantum corrections will affect more the LEOR than the HEOR mode in agreement with [8]. When using a TF density $n_0^{TF}(r)$, these corrections they turn out to be small, except for the typical divergent contribution very near the classical turning point at the Fermi energy. For a proper evaluation of quantum effects we have used the exact initial density for the determinantal ground state [12]. Due to the appearance of the density derivatives in Eq. (7), the richer the structure of $n_0(r)$ is, the greater the quantum correction will be. In other words, shell effects are essential for a correct description, and the use of a TF initial distribution might lead to erroneous results. The spatial structure of the transition density for the HEOR and LEOR given by Eq. (6), is not affected when taking into account the chosen

separable interaction. In Figs. 1-2 we show the transition density for a A=40 and A=224, using the exact and the TF initial distributions for both the high and low lying 3^- state. Quantum effects are clearly important in the low lying mode, and the implications of the neglected terms in the Vlasov approach are explicitly shown in the figures. The results are in qualitative agreement with RPA calculations using more realistic forces, and with the experimental observation of the transition density in ^{208}Pb [13]. A more detailed analysis of the quantum corrections to other fluid dynamical quantities and to the response will be given elsewhere.

In summary, we have shown that quantum corrections to the Vlasov approach arise from the dynamical evolution and from the initial distribution. Whereas the TF initial state approximation reasonably predicts the position of the resonances and their strength, it is unable to reproduce the structure of the experimental transition densities [13]. In this work we have discussed the origin of this failure and explicitly evaluated the quantum corrections to the Vlasov + TF approach for a specific model. We have shown the relevance of the initial distribution in the prediction of fluid dynamical quantities for low lying modes of high multipolarity.

The authors thank A.F.Guillermet and V.Grünfeld for a critical reading of the manuscript. One of us (LDS) wants to acknowledge for the grant of a research fellowship from CONICET, Argentina.

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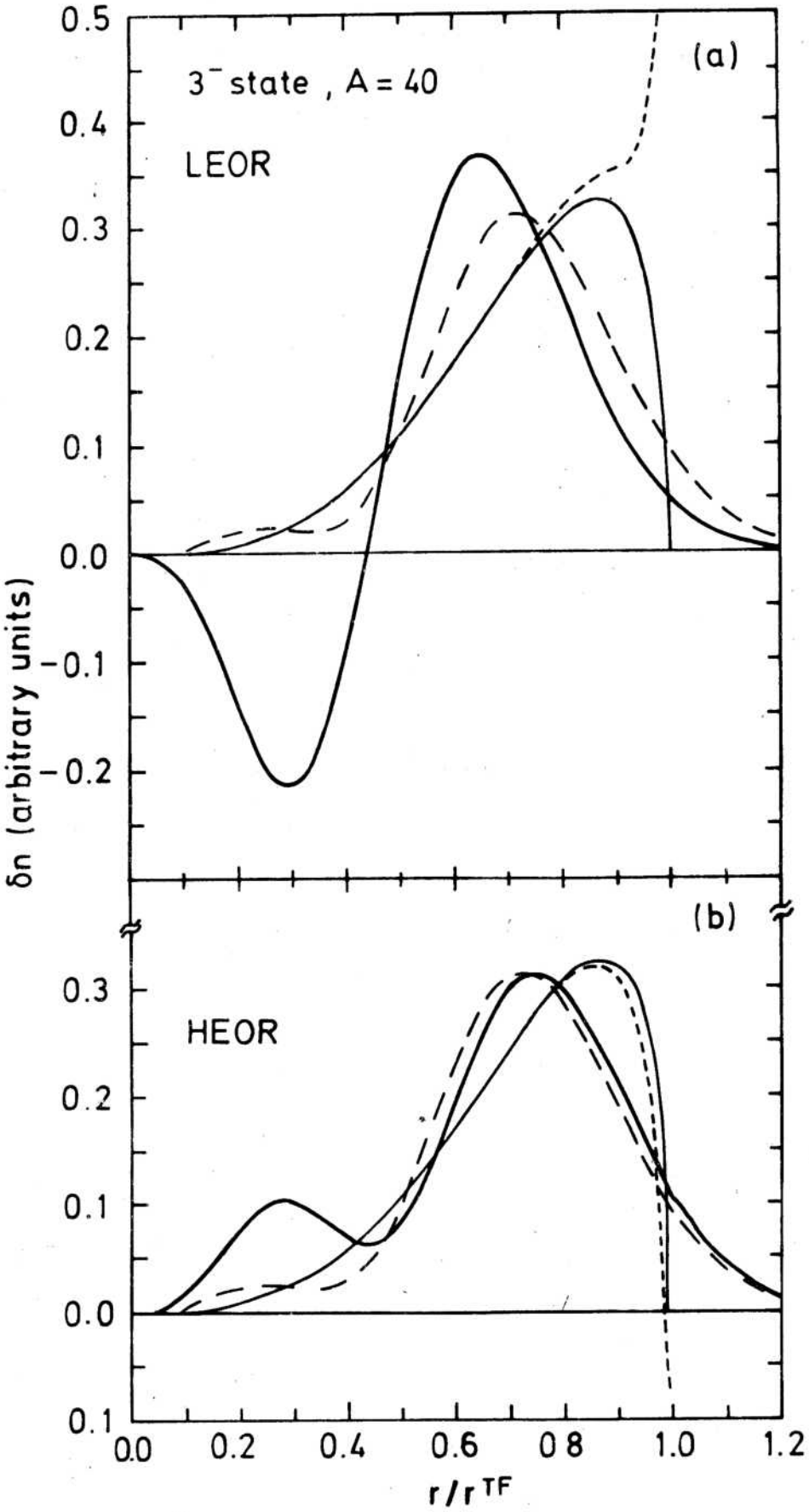
Figure Captions:

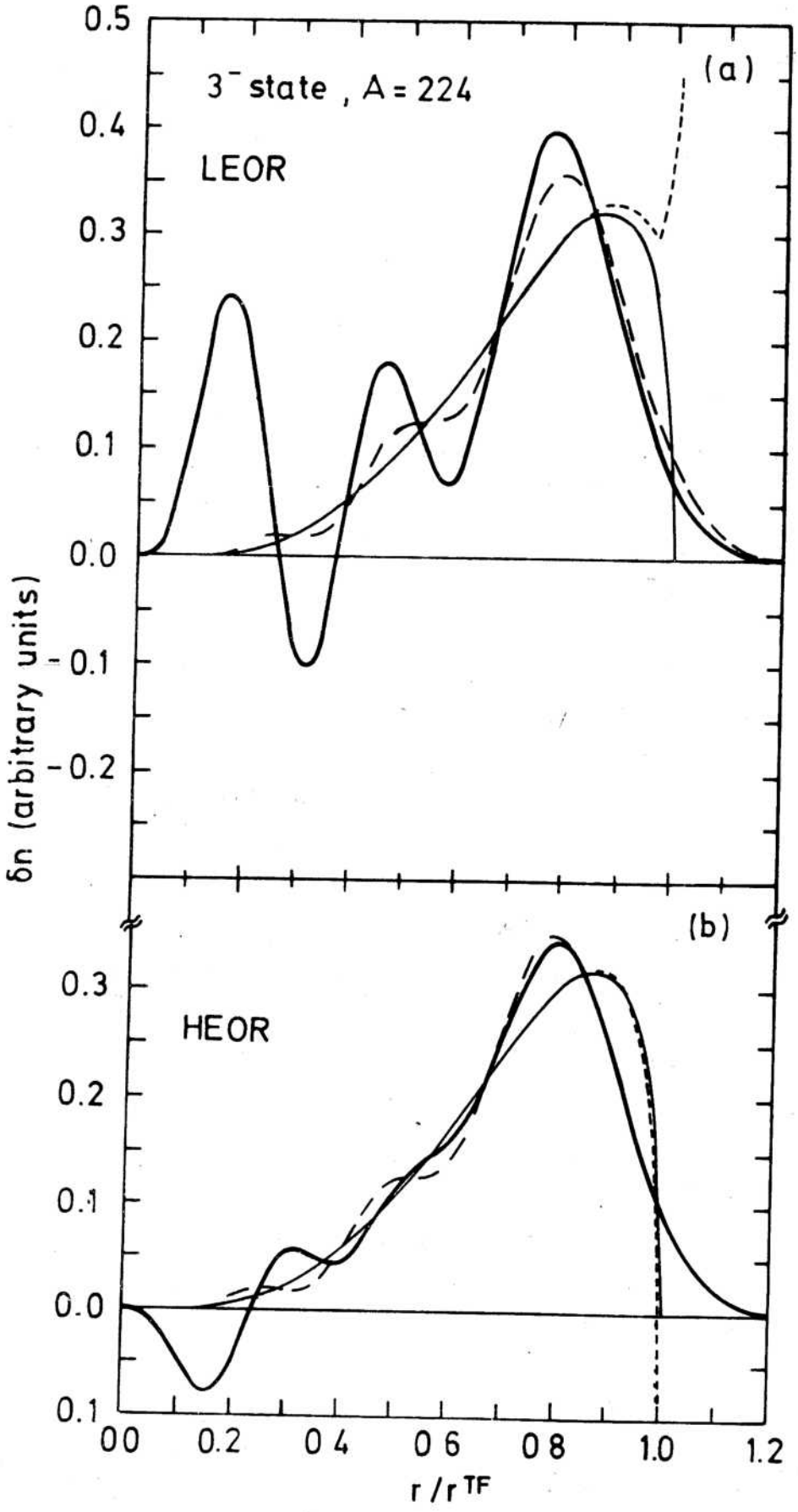
Fig. 1:

Spatial structure of the octupole transition density of a nucleus with A=40 particles. a) LEOR; b) HEOR. The results of Eq.(6) and its (semi)classical approximations are depicted for the following cases: (▬▬) Exact determinantal initial state (SD) + quantal dynamics ; (– – –) SD + classical dynamics; (——) TF + classical dynamics ; (-----) TF + quantal dynamics.

Fig. 2 :

Same as in Fig.1 for A=224.


3⁻ state , A = 40
(a)
LEOR
(b)
HEOR
δn (arbitrary units)
r/r TF
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
0.3
0.2
0.1
0.0
0.1
0.0
0.2
0.4
0.6
0 8
1.0
1.2


3⁻ state , A = 224
(a)
LEOR
(b)
HEOR
δn (arbitrary units)
r / r TF
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
0.3
0.2
0.1
0.0
0.1
00
0.2
0.4
0.6
0.8
1.0
1.2