MICROSCOPIC DESCRIPTION OF YRAST STATES IN SPHERICAL NUCLEI

R.J. LIOTTA
Research Institute of Physics, S 104 05 Stockholm 50, Sweden

and

C. POMAR
Research Institute of Physics, S 104 05, Stockholm 50, Sweden
and Departamento de Física, Comisión Nacional de Energía Atómica, 1429 Buenos Aires, Argentina

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The dynamical equations that control a many-body nucleus are derived in terms of correlated subsystems. The energies of states which can be described within one vector of the correlated basis are calculated. It is found that for yrast states those energies agree well with experimental data.

About twenty years ago De-Shalit proposed to describe low-lying states in even–odd nuclei closed to magic cores as the coupling of core excitations to single-particle states [1]. Since then, much work was done by many authors to analyse nuclear spectra in terms of correlated states [2–6]. These efforts were at least in part impelled by the difficulties associated with the standard shell-model description of complex spectra: the dimensions of the shell-model basis soon become very large and no reasonable calculation can be performed. Instead, using correlated basis the physical vectors are generally well described within a few basis vectors and, therefore, drastic truncations of those basis are possible. In particular, it was recently shown that even lead nuclei with, say, s nucleons outside the ^{208}Pb core can be analysed using a basis that couple very few 2 – and $(s-2)$ – correlated (physical) states [7]. One proceeds in several steps. First, the two-particle system is evaluated. Then the four-particle system is calculated in terms of the two-particle states evaluated in the previous step. Next one calculates the six-particle system using the two- and four-particle states evaluated in the first two steps. In ref. [7] only up to six particles were analysed. But the underlying idea of this multistep shell-model method (MSM) can be generalized to any number of particles, as will be shown in this letter. Thus, let $\{\alpha_s\}$ be a system with s particles outside a double magic core. As in ref. [7], we divide the system $\{\alpha_s\}$ into the two subsystems $\{\alpha_n\}$ and $\{\alpha_2\}$ with n and 2 particles, respectively, such that $s = n + 2$. A general wavefunction corresponding to the α_s-system is written as $|\alpha_s\rangle = P^+(\alpha_s)|0\rangle$ where the s-particle creation operator is given by ‡1

$$P^+(\alpha_s) = (1+\delta_{n2})^{-1} \sum_{\alpha_2\beta_n} Y(\alpha_2\beta_n;\alpha_s)\,(P^+(\alpha_2)\,P^+(\beta_n))_{\alpha_s} \qquad (1)$$

and $Y(\alpha_2\beta_n;\alpha_s) = (1+\delta_{\alpha_2\beta_n})\,X(\alpha_2\beta_n;\alpha_s)$. The factor $(1+\delta_{n2})^{-1}$ in eq. (1) takes care of the double counting in the summation for the case $n = 2$. The surplus factor 1/2 for $\alpha_2 = \beta_2$ is cancelled by the δ-function relating Y to the wavefunction amplitude X. In eq. (1) and throughout this paper we use the same symbols to denote states as well as the corresponding angular momenta.

‡1 Greek letters are used to label states. The corresponding number of particles are given as subindices. Thus γ_p labels a p-particle state.

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The shell-model equations corresponding to the α_s-system can be obtained, as usual, using the Tamm–Dancoff approximation (TDA). That is, one writes the commutator $[H, P^+(\alpha_2)\, P^+(\alpha_n)]$ in normal form and keeps only linear terms in P^+P^+ (or its single-particle equivalent). After this calculation has been performed one finds that the two-body interaction appears in combinations that allow one to write the s-particle dynamical equation in terms of the energies and wavefunctions of the two- and n-particle system. This property was first found by Ring and Schuck in three-particle systems [5,6] and was later used in the analysis of four- [6,7] and six- [7] particle nuclei. The important point in ref. [7] is that systems calculated in a given step are like building blocks to be used in later steps. This MSM feature greatly simplifies the formalism, since all the recoupling coefficients contained in the equations of a given step are not passed to later steps.

After some algebra, one finds that the TDA equation corresponding to eq. (1) is, for $s > 4$

$$
\begin{aligned}
&[W(\alpha_s) - W(\alpha_2) - W(\alpha_n)]\langle\alpha_s|(P^+(\alpha_2)\, P^+(\alpha_n))_{\alpha_s}|0\rangle \\
&= \sum_{\alpha_2'\alpha_n'} \Bigg(\sum_{\alpha_{n-2}} Y(\alpha_{n-2}\alpha_2';\alpha_n)\,[W(\alpha_n') - W(\alpha_2) - W(\alpha_{n-2})]\,\langle\alpha_n'|(P^+(\alpha_{n-2})\, P^+(\alpha_2))_{\alpha_n'}|0\rangle\, \hat{\alpha}_n\hat{\alpha}_{n'} \begin{Bmatrix} \alpha_2' & \alpha_{n-2} & \alpha_n \\ \alpha_2 & \alpha_s & \alpha_n' \end{Bmatrix} \\
&+ \frac{1}{2} \sum_{\beta_2\beta_4\gamma_2\alpha_{n-2}} Y(\beta_2\alpha_{n-2};\alpha_n)\,\langle\alpha_n'|(P^+(\gamma_2)\, P^+(\alpha_{n-2}))_{\alpha_n'}|0\rangle\, Y(\gamma_2\alpha_2';\beta_4)\,[W(\beta_4) - W(\alpha_2) - W(\beta_2)] \\
&\times \langle\beta_4|(P^+(\beta_2)P^+(\alpha_2))_{\beta_4}|0\rangle\, \hat{\alpha}_n\hat{\alpha}_n'\hat{\beta}_4^2 \begin{Bmatrix} \alpha_{n-2} & \beta_2 & \alpha_n \\ \alpha_2 & \alpha_s & \beta_4 \end{Bmatrix} \begin{Bmatrix} \alpha_{n-2} & \gamma_2 & \alpha_n' \\ \alpha_2' & \alpha_s & \beta_4 \end{Bmatrix} \Bigg) \langle\alpha_s|(P^+(\alpha_2')\, P^+(\alpha_n'))_{\alpha_s}|0\rangle\,, \qquad (2)
\end{aligned}
$$

where W is energy referred to the core and the wavefunction amplitudes Y are given by eq. (1). These quantities and the projections $\langle\alpha_r|(P^+(\alpha_p)\, P^+(\alpha_q))_{\alpha_r}|0\rangle$ needed to evaluate eq. (2), are calculated in previous steps. Considering that eq. (2) is valid for any value of n, one can well say that it is a simple equation.

As for the cases with six (or less) particles outside the core [5–7], eq. (2) is not hermitian and its dimension is larger than the corresponding shell-model dimension. This feature is a consequence of the violations of the Pauli principle as well as the overcountings that are present in the basis set of vectors $\{(P^+(\alpha_2)\, P^+(\alpha_n))_{\alpha_s}|0\rangle\}$. To correct these deficiencies one must evaluate the overlap matrix among the basis vectors (metric matrix). One can then use any of the available methods to describe a state within an overcomplete non-orthogonal set of basis vectors [4,7,8]. However, if a physical state is proportional to a basis vector only, one does not need to know the overlap matrix to calculate the energies given by eq. (2). As suggested by previous calculations [7], this would be the case for the yrast states (that we call θ_p), which are generally isolated from the rest of the p-particle spectrum. We thus assume

$$|\theta_s\rangle = X(gs_2\theta_n;\theta_s)\, P^+(gs_2)\, P^+(\theta_n)|0\rangle\,, \qquad (3)$$

where X now only plays the role of a normalization constant and gs_2 labels the two-particle ground state. The approximation (3) would be valid as long as the norm of the vector $|\theta_s\rangle$ is not negligible, i.e. as long as the Pauli principle is not very effective in blocking the state (3). But if the number of particles n is large enough the single-particle shells upon which the vector (3) is built would be filled and the approximation (3) would break down. This is what happens with the vector $P^+(\theta_2)\, P^+(gs_n)|0\rangle$ in ^{202}Pb (for details see p. 159 of ref. [7]). This vector is much less important than the vector (3) because the states $|\theta_2 \neq gs_2\rangle$ are very "pure" (i.e. they are described by very few shell-model configurations) in comparison with the normal pairing vibration $|gs_2\rangle$.

Introducing eq. (3) into eq. (2) and since $1 = X(gs_2\theta_n;\theta_s)\langle\theta_s|P^+(gs_2)\, P^+(\theta_n)|0\rangle$, one obtains $W(\theta_s) - W(gs_2) - W(\theta_n) = W_1 + W_2$, where $W_1 = (1 + \delta_{n4})\,[W(\theta_n) - W(gs_2) - W(\theta_{n-2})]$ and $W_2 = (1 + \delta_{n4})\,[W(gs_4) - 2W(gs_2)]$ correspond to the two terms in the rhs of eq. (2). Note that for W_2 the approximation (3) implies $|\beta_4\rangle = |gs_4\rangle$ and, therefore, the factor 1/2 in the last term of eq. (2) is cancelled by the factor 2 in $Y(gs_2gs_2;gs_4)$.

With $E(\theta_s) = W(\theta_s) - W(gs_s)$ one finally obtains

$$E(\theta_s) = E(\theta_n) + \Delta E(\theta_n), \qquad \Delta E(\theta_n) = E(\theta_n) - E(\theta_{n-2})\,, \qquad (4a, b)$$

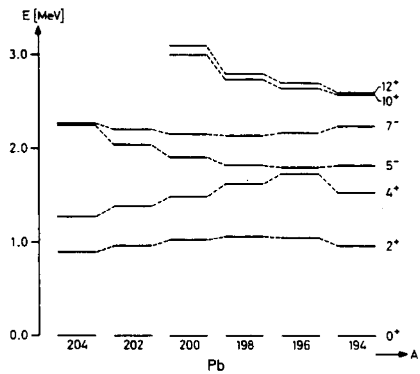

Fig. 1. Experimental neutron hole spectra in the lead region.

i.e. the energy of the states θ_s would follow a monotonous variation with the number of particles n. If ΔE is negative (positive) in the beginning of the major shell $E(\theta_s)$ would decrease (increase) monotonously with the number of particles. This is indeed a tendency followed by all spherical nuclei. As an example (which is rather typical) we show in fig. 1 the hole spectra in the lead region. Other examples are shown, e.g. in ref. [9].

As mentioned above, eqs. (4) cease to be valid when the single-particle shells that constitute the vector (3) are filled. For instance, the states 4^+ in fig. 1 are built mainly upon the shells $p_{1/2}$, $p_{3/2}$, $f_{5/2}$. These shells are filled in ^{194}Pb and therefore the tendency mentioned above is not followed in this nucleus. One may then apply eq. (3) starting from the other extreme of the major shell. Thereby the meaning of "empty" and "full" shells are interchanged and eqs. (4) continue to be valid, in agreement with experimental data.

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