Critical behavior of Young's modulus for two-dimensional randomly holed metallized Mylar

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We have measured the conductance and Young's modulus of sheets of randomly holed metallized Mylar. The percolation threshold is $p_c=0.34\pm0.02$, the conductivity exponent is $\bar{t}=1.4\pm0.2$, and the elastic exponent, $\bar{f}=5.3\pm0.7$, is at least $\frac{3}{2}$ larger than that for lattices. This confirms Halperin, Feng, and Sen's prediction [Phys. Rev. Lett. **54**, 2391 (1985)].

The behavior of a system near the percolation threshold can be described by a few critical exponents which, according to generalized intuition, should be independent of the detailed way in which randomness is introduced. In particular, whether the system is a lattice or continuum should be irrelevant when the correlation length is much larger than the lattice parameter for the former. But, as Gawlinski and Stanley[1] wrote: "Intuitive arguments can be misleading, however, and in percolation one is often confronted with facts that may contradict intuitive guesses." The elastic moduli of a system undergoing percolation have been considered as likely candidates to show such contradictions.[2-4] Indeed, Halperin, Feng, and Sen[5] recently used a scaling analysis to estimate various critical exponents in so-called "Swiss-cheese models:" equal, randomly centered spherical holes are scooped out of a conducting elastic solid. The fact that these models present a continuous distribution of bond strengths (if we use the word "bond" for the isthmus between holes, which can be reduced by new holes) has rather subtle consequences on the critical exponents. While those for geometrical properties like the size of the infinite cluster or the correlation length are not affected,[1,6] the exponents for the conductivity and for Young's modulus in three dimensions were predicted by Halperin *et al.* to exceed their lattice counterparts by at least $\frac{1}{2}$ and $\frac{5}{2}$, respectively. In two dimensions, they expected the differences to be less marked: while the elastic exponent $\bar{f}$ for Young's modulus should be at least $\frac{3}{2}$ larger than that for lattices, the conductivity exponent $\bar{t}$ (also known as $\bar{\mu}$) should remain unchanged.

In this note we report measurements of conductivity and Young's modulus that we carried out on two-dimensional randomly holed metallized Mylar in order to test Halperin *et al.*'s predictions. Our results confirm those predictions in two dimensions.

Our samples were 10×11-cm^2 sheets of 0.075-mm-thick metallized Mylar foil. This material presents two advantages for our purpose: the thinness of the metallic layer on the Mylar makes the measurement of conductance a simple affair; and holes can be (rather) easily punched in Mylar without having to flatten any puckering later. On the other hand, Mylar is a polymer and its elastic properties are different from those of metals, but its load-displacement curve is satisfactorily linear for small deformations. As a polymer's elastic moduli are very temperature dependent, we took care to measure always at the same (room) temperature. Of course, the fact that the elastic material is an insulator while the conductor has negligible stiffness presents no difficulty, since they are in parallel.

We punched 4-mm-diameter holes at random on a 10×10-cm^2 square centered on the Mylar sheets. This was our sample proper: the strips at both ends, 0.5×10 cm^2, were held by clamps. The sample hung freely from the upper clamp, which was made of aluminum and was fixed to a sturdy iron bracket. The lower end of the clamp was held by a light clamp made of two strips of phenolic resin 10.5 cm long, 12 mm wide, and 3 mm thick. We cut two facing grooves along the inner faces of these strips, 2 mm from the upper edge, and laid a piece of stripped copper wire along one of the grooves before sandwiching the Mylar foil between the halves of the clamp. This wire locked the foil in place, and provided an electrical contact which extended across the end of the sample. We tightened each clamp with eight small brass screws and nuts. The ends of the copper wire protruded from the ends of the lower clamp, and were the current and voltage leads. At the upper clamp, it was enough to fix the leads to the tightening screws. We drilled a row of small holes through the lower clamp, which we used to hang the loads from. Near the percolation threshold the lower clamp showed a marked tendency to rotate, and the point of application of the load had to be varied slightly to keep it horizontal.

In order to place the holes in the Mylar foil as randomly as possible, we drew 10×10-cm^2 cards with a desk-top computer, on which it marked groups of hole centers using the standard pseudorandom number generator, and we used these cards as guides for our punch. We gathered the Mylar plugs and weighted them in a good precision balance. We consider that our p values (fraction of remaining material) have no error compared to the other magnitudes we measured.

We measured the conductance of the sheets by passing a current, typically 0.05 A or smaller, and measuring the voltage drop between clamps. These measurements were very reproducible, and their error is negligible. To measure Young's modulus we hung weights from the lower clamp and measured the resulting elongations with a cathetometer. This allowed a precision of 0.01 mm.

After measuring Young's modulus, $E(p)$, for the intact sheets ($p=1$), which demanded loads of several kilograms, we retightened both clamps and observed no slippage later, at much smaller loads. In Fig. 1 we show a

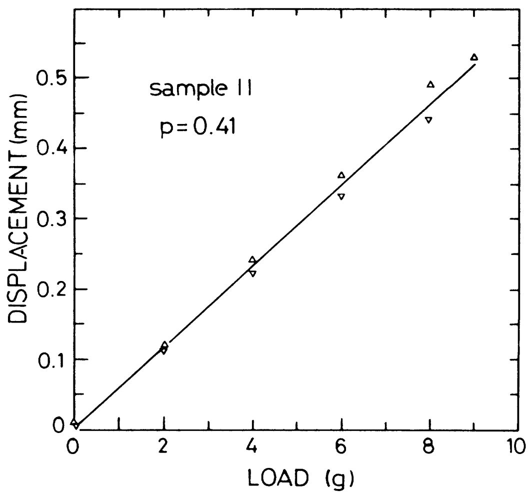

FIG. 1. Load-displacement curve for sample II at $p=0.41$. Load increasing, $\triangle$; decreasing, ∇. The line is the average of both sets of points, and gives Young's modulus.

load-displacement curve for sample II at $p=0.41$. Observe that the total elongation is 0.5 mm, that is, a 0.5% deformation. A small amount of hysteresis can be seen, which seemed to come mostly from rotations of the lower clamp due to the uneven loading: careful repositioning of the load along the clamp reduced this hysteresis. As we were unable to totally avoid it, part of it might well come from the sample itself. In any case, this effect was smaller for higher p values. It is not an instrumental error: the cathetometer carriage was displaced always in the same direction.

Figure 2 shows Young's modulus and conductance for sample II versus p. Both have been normalized by their $p=1$ values, and are dimensionless quantities. The expected error bars are smaller than the size of the points we have used: The scatter in the points comes mostly from the statistical fluctuations in the disposition of the holes. The results for sample I are quite similar, and would only clutter the figure.

Near the percolation threshold the conductivity is expected to behave as

$$\sigma \sim (p-p_c)^{\bar{t}} .$$

In Fig. 3 we plot $\sigma^{1/\bar{t}}$ vs p for sample II. We have chosen the exponent $\bar{t}$ which gives the best linear fit at low p. We have drawn this straight line, which intersects the p axis at p_c. We find for sample I

$$\bar{t}=1.3 \pm 0.2$$

and for sample II

$$\bar{t}=1.4 \pm 0.2 .$$

These values agree well with the known t for lattices,[7(a)] and a continuum.[7(b)] In particular, their agreement with recent values for Mylar sheets[7(b)] can be taken as a calibration of our apparatus. For both samples, the percolation threshold gives

$$p_c = 0.34 \pm 0.02 ,$$

an agreement which is surely fortuitous. This value also agrees with what is known of numerical simulations.[1,8–12] We estimated the errors quoted by removing at random half the points in each, and recalculating in each case both $\bar{t}$ and p_c. The relation between the number of holes, N, and the expectation value of p is $p=\exp(-N\pi\rho^2/A)$, where ρ is the radius of the holes and A is the area of the sample. The "critical" number of holes for our samples would then extrapolate to 890 ± 6. Our measurements went to 650 holes for each sample.

The method we have used above is the generally accepted one to find exponents when the critical p is not known, but it is rather inaccurate when the exponent is larger

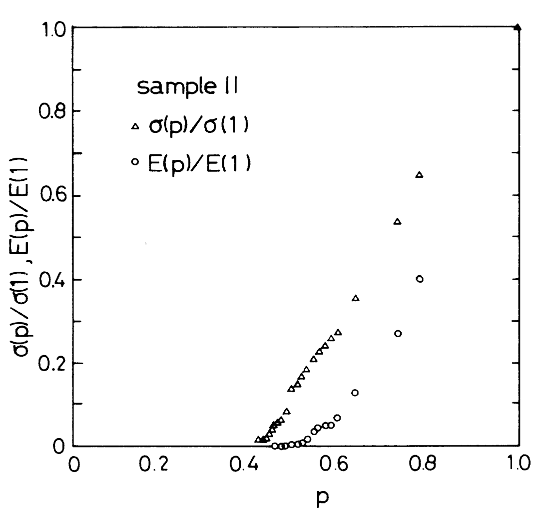

FIG. 2. Normalized conductance ($\triangle$) and Young's modulus ($\circ$) for sample II vs fraction of remaining material p.

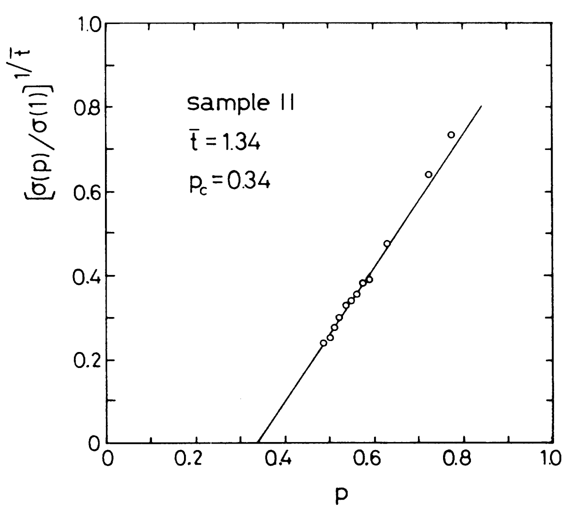

FIG. 3. Normalized conductance to the power $1/t$ vs p for sample II. Low points, affected by finite-size effects, have been deleted.

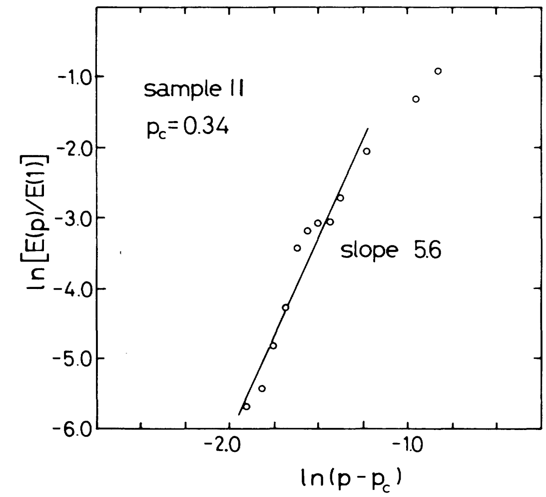

FIG. 4. Plot of $\ln[E(p)/E(1)]$ vs $\ln(p-p_c)$ for sample II. The critical p_c has been taken from Fig. 3, and is $p_c=0.34$.

than, say, three. This is the case for the expected value of $\bar{f}$, the elastic exponent. In addition, our measurements of Young's modulus are inherently noisier than those of the conductance. We shall therefore consider that p_c is well determined by the conductance for each sample, and calculate the exponent $\bar{f}$ from the slope of $\ln E(p)$ vs $\ln(p-p_c)$, shown in Fig. 4 for sample II. This procedure gives for sample I

$$\bar{f}=5.0\pm1.0$$

and for sample II

$$\bar{f}=5.6\pm0.8 \ .$$

The error bars correspond to using the extreme values of p_c (0.32 and 0.36) for p_c in $\ln(p-p_c)$.

The reader will have noticed that the lowest points in Fig. 2 do not appear in the figures that follow it. We have not used them in the calculation of critical exponents because they are affected by finite-size effects. The correlation length near p_c is $\xi(p)\simeq\xi_0(p-p_c)^{-\nu}$ with[1] $\nu\simeq\frac{4}{3}$. If we take $\xi_0\simeq4$ mm, the diameter of the punched holes, then ξ equals the side of the sample when $p-p_c\simeq0.09$, and points within this range should be suspect at least. This is the same criterion used by Sieradzki and Rong Li.[13] The points within this range show more scatter, and fall markedly below the straight line in Fig. 4. If we had included these points in our conductivity curves, they would have led to lower values of p_c, with larger error bars, and hence to higher values of the conductivity exponent. Both effects would be contrary to what is known. The use of a lower p_c in Fig. 4, and its equivalent for sample I, would have moved the suspect points further to the right: Inclusion of these points would have yielded higher values of the elastic exponents. By neglecting them, any error we might be committing goes against Halperin *et al.*'s prediction. Form all the above we think it is very unlikely that we have underestimated our error bars, and the elastic exponent is

$$\bar{f}=5.3\pm0.7 \ ,$$

taking into account both measurements. In Table I we have listed values of the elastic exponent for two-dimensional lattices. The value we have obtained for our two realizations of the Swiss-cheese model is clearly different from those in the table, and is approximately $\frac{3}{2}$ higher than them. In consequence, we conclude that Halperin *et al.*'s[5] prediction is right in two dimensions.

TABLE I. Elastic exponent f for two-dimensional lattices.

Author	Method	f
Kantor and Webman (Ref. 3)	Theoretical	3.6
Sieradski and Rong Li (Ref. 13)	Experimental	3.1 ± 0.1
Benguigui (Ref. 14)	Experimental	3.5 ± 0.4
Feng *et al.* (Ref. 15)	Numerical simulation	3.3 ± 0.5

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