Consistent Chiral Quantum Electrodynamics

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Abstract

We construct a unitary and renormalizable quantum field theory in 3+1 dimensions describing the interaction of chiral massless fermions with massive or massless photons.

I. Introduction

Some time ago the possibility of consistently quantizing chiral gauge theories (CGT) without adjusting its fermionic content was suggested[1]. A two dimensional example of this situation was provided by the Chiral Schwinger Model[2] (CSM). Many quantization methods were proposed for the CSM, some of them are quite general and not restricted to a two dimensional theory. However, we do not know of any of these methods that, when applied to a 3+1 dimensional CGT, lead to a unitary and renormalizable quantum field theory (QFT). Given this situation we follow a more pragmatic approach. We consider 3+1 dimensional chiral electrodynamics (CED) and ask the following question. Is it possible to write down a functional integral involving the CED lagrangian such that one obtains a consistent (unitary and renormalizable) QFT, describing the interaction of chiral massless fermions and photons? The answer to this question is affirmative. We do not know, however, whether the theory presented here is the only answer to the question made above. Some other work in this direction has already appeared in the literature[3,4].

The theory studied here is the result of several attempts to match a chiral interaction with unitarity and renormalizability. We have learned from these attempts that the conditions of unitarity and renormalizability impose severe restrictions on the possible theories.

This paper is organized as follows. In section II we describe the theory and discuss its renormalizability and unitarity. Section III presents our conclusions.

II. Description of the theory

The lagrangian density of this theory is,

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mu^2 A_\mu A^\mu - \frac{\lambda}{2}(\partial.A)^2 + \bar{\psi}(i\partial + eAP_-)\psi$$

$$-\frac{1}{2}\varphi\,\Box\,(\Box+\tilde{\mu}^2)\varphi - \varphi\,[(\Box+\tilde{\mu}^2)\,\partial.A - \frac{1}{2}G(A)]$$

$$+\frac{1}{2}\theta\Box\,(\Box+\tilde{\mu}^2)\theta - \theta\,[(\Box+\tilde{\mu}^2)\,\partial.A + \frac{1}{2}G\,(A)] \quad , \tag{2.1}$$

where,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ,$$

$$G(A) = \frac{e^3}{32\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,F_{\mu\nu}F_{\rho\sigma} = \frac{e^3}{16\pi^2}\,F_{\mu\nu}\tilde{F}^{\mu\nu} \quad ,$$

$$P_- = \frac{1-\gamma 5}{2} \quad , \tag{2.2}$$

our notations for the Dirac lagrangian density and the γ matrices are the same as those of ref.(5).

The perturbation theory arising from *(2.1)* is renormalizable by power counting. The superficial degree of divergence of a diagram G is given by,

$$\omega(G) = 4 - \frac{3}{2}E_F - E_A - \delta \quad , \tag{2.3}$$

where E_F is the number of external fermion lines, E_A is the number of external lines of the field A_μ and δ is the number of derivatives that act on the external lines. Note that δ will be increased when the number of external φ-legs or θ-legs are increased. Therefore, there is a finite number of primitively divergent diagrams. It is important to note that this would not be the case if the free propagators for the fields φ and θ would decrease slower than k^{-4} for large momentum k, or, what is the same, if the free actions for these fields would have a number of derivatives less than four.

Now we consider the symmetries. This theory possesses three BRST nilpotent symmetry transformations (ST) given by,

$$
\begin{array}{lll}
\mathrm{BRST}_1 & \mathrm{BRST}_2 & \mathrm{BRST}_3 \\
\delta A_\mu=\xi\partial_\mu c & \delta A_\mu=0 & \delta A_\mu=0 \\
\delta\varphi=-\xi c & \delta\varphi=\xi d & \delta\varphi=\xi b \\
\delta\theta=\xi c & \delta\theta=\xi d & \delta\theta=\xi b \\
\delta\bar{c}=i\xi\partial.A & \delta\bar{d}=i\xi[2\partial.A+\Box(\varphi-\theta)] & \delta\bar{b}=i\xi[(\Box+\tilde{\mu}^2)[2\frac{\partial.A}{\Box}+\varphi-\theta]] \\
\delta c=0 & \delta d=0 & \delta b=0 \\
\delta\varphi=i\xi eP\_\psi & \delta\psi=0 & \delta\psi=0 \\
L_G=-i\lambda\bar{c}(\Box+\frac{\mu^2}{\lambda})c & L'_G=-i\bar{d}(\Box+\tilde{\mu}^2)d & L''_G=-i\bar{b}\Box b \\
\bar{c}=\bar{c}^\dagger \quad c=c^\dagger & \bar{d}=\bar{d}^\dagger \quad d=d^\dagger & \bar{b}=\bar{b}^\dagger \quad b=b^\dagger
\end{array}
\tag{2.4}
$$

where below each ST we have written the corresponding uncoupled ghost action and the hermiticity assignment for the ghost fields. The BRST_1 transformation is not a ST of $L+L_G$ but is a ST of the functional integral that arises from *(2.1)*. Indeed, neglecting total derivatives, the variation of *(2.1)* under BRST_1 is given by,

$$\delta_1(L+L_G) = -\ \xi c\ G(A) \tag{2.5}$$

that cancels the variation of the fermionic measure of integration[6]. We should note also that $BRST_1$ as it stands in *(1.4)* is not strictly nilpotent, but only over the equations of motion. However introducing[7] appropriate auxiliary fields as in ref.(8) one obtains a strictly nilpotent transformation.

Let us consider now the unitarity of this theory. In order to do this we should first know which are its asymptotic states. We do perturbation theory on the coupling constant e, so that the free lagrangian is obtained replacing $e=0$ in *(2.1)*. This free lagrangian is diagonalized by the following combinations of the original fields,

$$\begin{aligned} \varphi' &= \varphi + \frac{\partial . A}{\Box} \quad , \\ \theta' &= \theta - \frac{\partial . A}{\Box} \quad , \end{aligned} \tag{2.6}$$

the free lagrangian density in terms of θ' and φ' reads,

$$L = -\frac{1}{4} F^2(A) + \frac{1}{2}\mu^2 A^2 - \frac{\lambda}{2}(\partial . A)^2 + i\bar{\psi}\, \partial\psi - \frac{1}{2}\, \varphi'\Box(\Box+\tilde{\mu}^2)\varphi' +$$

$$+\frac{1}{2}\, \theta'\Box\, (\Box+\tilde{\mu}^2)\theta' \tag{2.7}$$

hence we have the following one particle asymptotic states, depending on whether $\mu^2=0$ or $\mu^2>0$. If $\mu^2=0$, we have for the A_μ-sector two photon states with polarization vectors orthogonal to its 3-momentum, the physical ones with positive norms (PN). A state with negative norm (NN) and polarization vector equal to its 4-momentum and a PN state with polarization vector orthogonal to the other three. If $\mu^2>0$ we have three PN massive states with polarization vectors orthogonal to its 4-momentum and mass equal to μ^2, and a NN state with polarization vector equal to its 4-momentum and mass equal to μ^2/λ. For the φ'-sector we have, due to its higher derivative action[9], two states one of PN with mass equal to $\tilde{\mu}^2$ and the other with NN and vanishing mass. For the θ'-sector, we have the same situation as for the φ'-sector, but with the masses interchanged, that is, a PN one with vanishing mass and a NN one with mass equal to $\tilde{\mu}^2$. Of course in both cases the PN fermion states are present.

Our discussion of the unitarity of this theory relies on the formalism presented in ref.(8). The physical states are defined as the ones that are annihilated by the three BRST charges corresponding to the three BRST transformations $BRST_i$ $(i=1,2,3)$. Regarding this point it is worth noting that these three transformations commute with each other, so that no contradiction is involved in the above definition of physical states.

An analysis of the algebra of the creation and annihilation operators of these states between themselves and with the three nilpotent charges leads to the following classification of states[7]. We obtain three quartets with the following members. One quartet involves the NN state of the A_μ sector, the auxiliary field state if $\mu^2 > 0$, the 3d-longitudinal state if $\mu^2=0$ and the ghost states corresponding to c and $\bar{c}$. The second quartet involves the NN massless state of the φ'-sector, the PN massless state of the θ'-sector and the ghost states corresponding to b and $\bar{b}$. The other quartet is formed by the NN state of the φ'-sector with mass $\tilde{\mu}^2$, the PN state of the θ'-sector with the same mass, and the ghost states corresponding to d and $\bar{d}$.

Therefore the only physical states that remain are the fermion states with right component $(P_+\psi)$ decoupled, the two 3-d transverse massless photons if $\mu^2=0$ and the three 4-d transverse massive photons if $\mu^2>0$. We remark that the unitarity of the S-matrix in the above mentioned physical states can also be shown[7)] directly from the perturbative series of this theory using Cutkovsky rules and Ward identities[10)]. In this way one obtains a unitary and renormalizable field theory. Regarding this last point we mention that a detailed analysis of the arbitrary constants compatible with the Ward identities appearing in this theory only leaves the photon mass and the coupling constant e as free parameters[7)]. With respect to the other two parameters λ and $\tilde{\mu}^2$ that appear in (2.1), one can show[7)] that the S-matrix is independent of these parameters. This is essentially so because adding to the Lagrangian (2.1) BRST-variations of properly chosen terms[11)] one can change the values of λ and $\tilde{\mu}^2$ without affecting the boundary conditions.

In order to see in a specific example in terms of diagrams how the anomaly is effectively cancelled in this theory we have included fig. 1.

An interesting feature of this theory arises when looking at the effect of integrating out the fields φ and θ. In that case one obtains the following non-local lagrangian density,

$$L = -\frac{1}{4}F^2(A) + \frac{1}{2}\mu^2A^2 - \frac{\lambda}{2}(\partial.A)^2 + \bar{\psi}\,(i\partial+eAP-)\psi - \partial.A\,\frac{1}{\Box}\,G(A) \quad , \tag{2.8}$$

this theory remains invariant under $BRST_1$. This symmetry leads to a unitary theory in much the same way as in the local version.

The number of primitively divergent diagrams is also finite as in the local case. The parallel of fig. 1 in the local version is given by fig. 2.

Another interesting point is the consideration of radiative corrections to the anomaly. In ref. (12) it is shown that these corrections exist, however this is an effect where the triangle itself generates the correction so that due to the general case of the identities of figs. 1 and 2 those corrections do not appear in this theory. How this works is shown diagrammaticaly in fig. 4 for the non-local version.

III Concluding remarks

As we can see from figs. 1 and 2, this theory provides a cancellation mechanism for the anomaly, different from the adjustment of the fermion content of the theory. It is important, however, to realize that there is a big difference between both procedures. The adjustment of the fermionic content can cancelthe anomaly by means of the introduction of additional physical particles. The mechanism presented here cancels the anomaly by means of properly coupled ghost fields, that is fields that do not lead to physically observable states.

Although we have not dealt here with a first principle justification of the present quantization of CED, we believe that an answer to this question is certainly a desirable goal. It is clear that a generalization of a Lagrangian density like *(2.1)* to other anomalous theories is not at all obvious.

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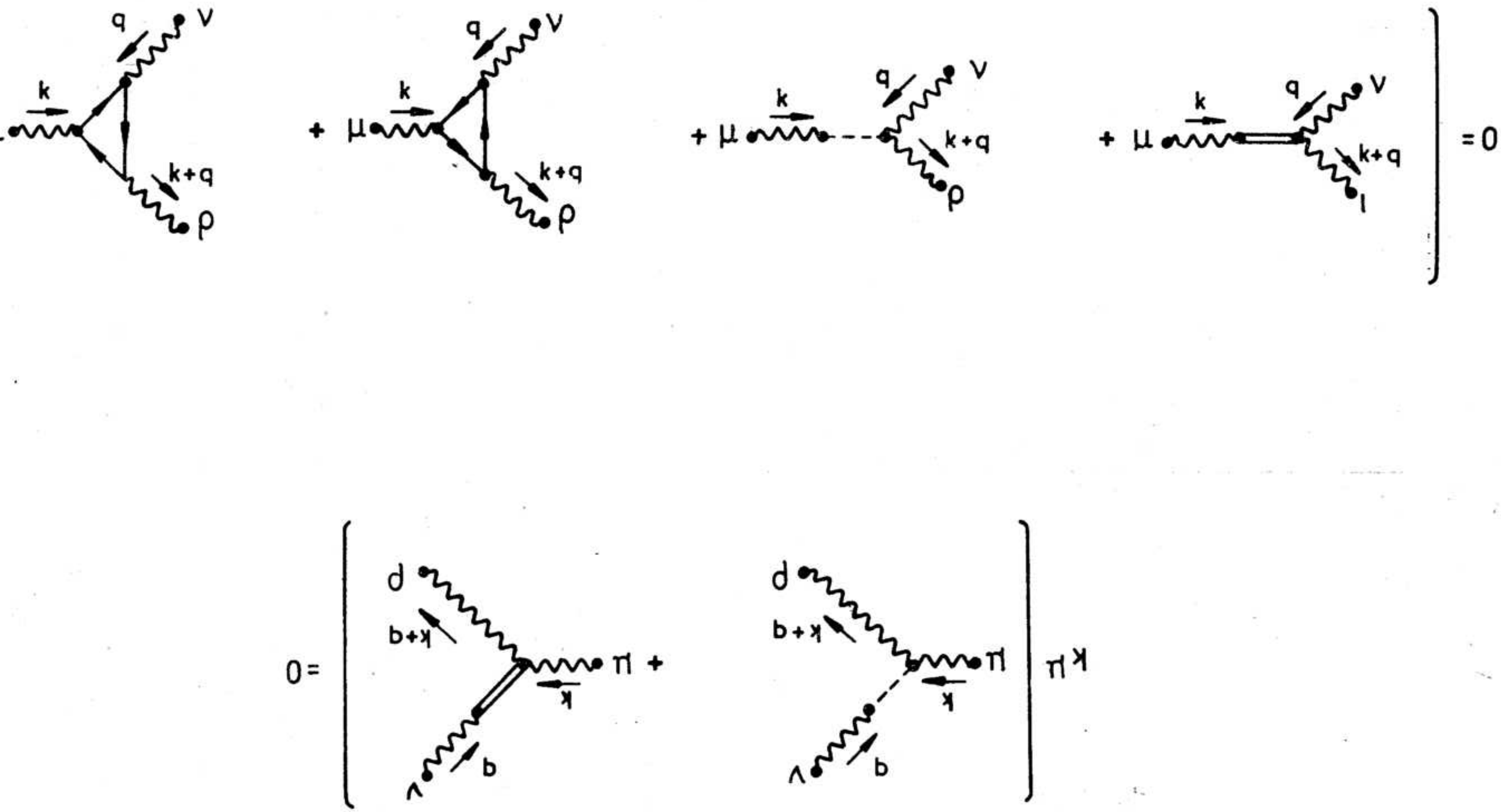
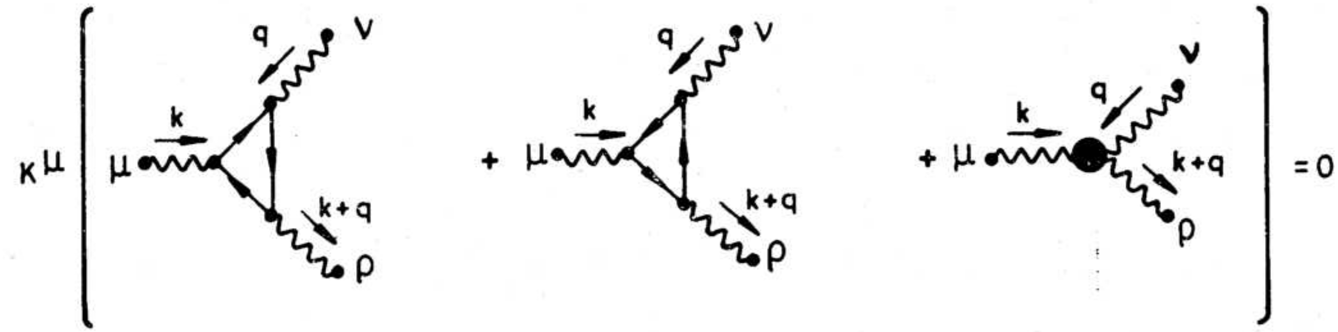
Figure Captions

Fig. 1: Example of cancellation of anomalies in the local version of this theory, as given by *(2.1)*. The solid lines denote fermion propagators, the wavy lines denote photon propagators, the dashed lines denote φ-propagators, and the double lines denote θ propagators.

Fig. 2: The same example as in fig. 1 but in the non-local version of this theory, as given by *(2.8)*. The big blob denotes the non-local vertex appearing in (2.8).

Fig. 3: Cancellation of radiative corrections to the anomaly.

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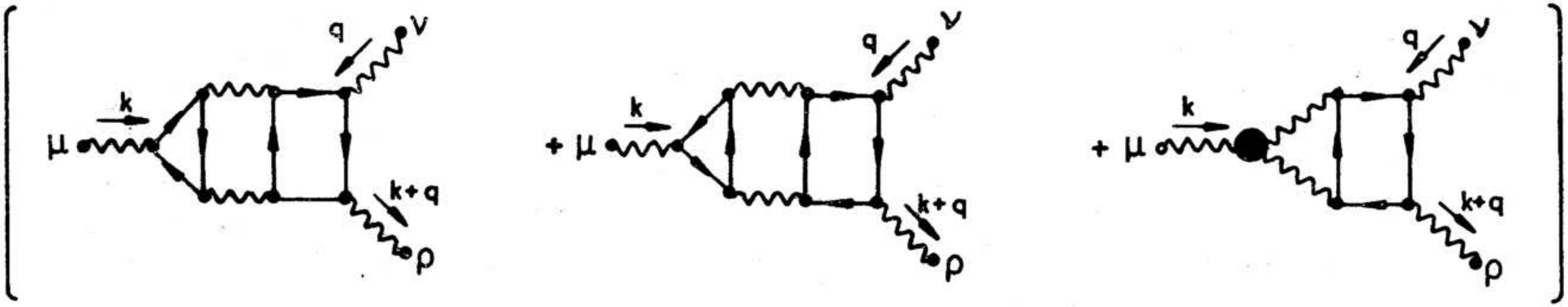
k^{μ}  $= 0$