APPLICATION OF THE STATISTICAL DESCRIPTION OF TWO-PHASE FLOWS TO INTERFACIAL AREA ASSESSMENT

G. Guido-Lavalle and A. Clausse

Div. Termohidráulica - Centro Atómico Bariloche
8400 Bariloche. Río Negro. ARGENTINA

SUMMARY

Gas-liquid two-phase flows are described by means of a bubble distribution function. The steady-state case is numerically solved and the results show that the interfacial area density increases, have a maximum at an intermediate value and then decreases with void fraction. This behavior is considered to be due to the different flow regimes: bubbly, slug and annular that appear with increasing void fraction.

1. INTRODUCTION

The interfacial area density is one of the most important variables in two-phase flows since it governs the mass, momentum and energy transfer between phases. It is also expected that a mechanistic model capable of predicting interfacial area density given the macroscopic parameters, should also be capable of predicting flow pattern transitions.

Such models cannot be derived from a macroscopic description of the flow since during the averaging procedure the information about the bubble volume distribution is lost. This, in turn, yields to the necessity of introducing a specifically formulated closure law for the interfacial area density. The model cannot either be based on a molecular microscopic description, for such a calculation is impracticable in most situations.

In this paper a statistical mechanical treatment of gas-liquid flows is presented, which gives an intermediate degree of detail in the description of the flow. This method can be called a "mesoscopic" description, and it has been widely used in particle-fluid two-phase flows [1-3] and some work has also been done in gas-liquid flows [4]. This formulation leads to a bubble distribution function transport equation which is presented in next section. Herein the spatially-homogeneous steady-state problem is numerically solved and its results are discussed.

2. THE TRANSPORT EQUATION

A bubbly gas-liquid flow is described herein by the distribution function $f(\mathbf{v},\mathbf{x},V,t)$, being $f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV$ the number of bubbles per unit volume at time t at position $\mathbf{x}$ with volumes between V and $V+dV$, and velocities between $\mathbf{v}$ and $\mathbf{v}+d\mathbf{v}$. As it contains much more information than the macroscopic variables, these last can be obtained as moments of the distribution function:

- *Number Density* (number of bubbles per unit volume):

$$N'''(\mathbf{x},t) = \iint f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV \qquad (1)$$

where the integrals must be evaluated over all possible volumes and velocities.

- *Interfacial Area Density* (total interfacial area of the bubbles per unit volume):

$$A(\mathbf{x},t) = \iint a(V)\, f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV \qquad (2)$$

where the function $a(V)$ gives the area of a bubble of volume V. If the bubbles are spherical this function is:

$$a(V) = (36\,\pi)^{1/3}\, V^{2/3} \qquad (3)$$

- *Void Fraction* (volume of gas per unit volume):

$$\alpha(\mathbf{x},t) = \iint V\, f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV \qquad (4)$$

- *Mean Bubble Volume*

$$\bar{V} = \frac{\iint V\, f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV}{\iint f(\mathbf{v},\mathbf{x},V,t)\,d\mathbf{v}\,dV} = \frac{\alpha(\mathbf{x},t)}{N'''(\mathbf{x},t)} \qquad (5)$$

The evolution of the system is given by its initial state, the imposed boundary conditions, the interaction between bubbles and the interaction between the bubbles and the surrounding liquid.

Considering only binary interactions between bubbles, a bubble may divide into two smaller ones, and two bubbles may coalesce creating a bigger one. Therefore it is possible to write the evolution equation:

$$\frac{D}{Dt} f(\mathbf{v},\mathbf{x},V,t) = B(\mathbf{v},\mathbf{x},V,t) + C(\mathbf{v},\mathbf{x},V,t) + S(\mathbf{v},\mathbf{x},V,t) \qquad (6)$$

where B and C give the rate of variation of f due to breakage and coalescence respectively, and S is the source term. The derivative of the left hand side includes derivations in all the variables, and is written as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}\,\nabla_{\mathbf{x}} + \frac{\mathbf{F}}{m}\,\nabla_{\mathbf{v}} + \frac{dV}{dt}\,\frac{\partial}{\partial V} \qquad (7)$$

where $\mathbf{F}$ is the sum of the external forces acting on a bubble and m its mass.

The bubble breakup probability is assumed to be proportional to the distribution function. The probability of coalescence is considered proportional to the product of the distribution functions of both bubbles at a given spatial location [5]. Consequently,

the breakage function B is given by:

$$B(v,\mathbf{x},V,t) = \int\int\int\int \Big[b(vV,v''V''|v'V')\, f(v',\mathbf{x},V',t) - b(v'V',v''V''|vV)\, f(v,\mathbf{x},V,t) \Big] dv'dv''dV'dV'' \quad (8)$$

The first term in the right-hand side corresponds to the gain of bubbles due to breakage of bigger bubbles and the second term gives the loss due to their breakage in smaller bubbles.

The kernel $b(vV,v''V''|v'V')$ is the probability that a bubble of volume V' and velocity v' splits into two bubbles of volumes V and V'' and velocities v and v'' respectively. Herein, the kernel is assumed to be independent of the position or time. Moreover, the mass and total volume of gas is conserved, regarding that the gas density is constant. It is not required in this model that the momentum or the kinetic energy are conserved, because during the breakage or coalescence process momentum and energy are interchanged with the liquid. Thus,

$$b(vV,v''V''|v'V') = \beta(vV,v''V''|v'V')\ \delta(V+V''-V') \quad (9)$$

where δ is the Dirac delta function and β is the kernel without the volume conservation requirement. Furthermore, the symmetry condition must be fulfilled:

$$b(vV,v''V''|v'V') = b(v''V'',vV|v'V') \quad (10)$$

Following similar balance arguments, the coalescence function is:

$$C(v,\mathbf{x},V,t) = \quad (11)$$

$$= \int\int\int\int \Big[c(vV|v'V',v''V'')\, f(v',\mathbf{x},V',t)\, f(v'',\mathbf{x},V'',t) - c(v''V''|vV,v'V')\, f(v',\mathbf{x},V',t)\, f(v,\mathbf{x},V,t) \Big] dv'dv''dV'dV''$$

where the coalescence kernels are similar to the breakage kernels and have to satisfy similar conditions.

The resulting equation is similar to what is called the generalized Boltzmann transport equation in kinetic theory [6,7], with the difference that here each bubble has an additional internal coordinate, V.

The interaction between the surrounding liquid and the bubbles is a distinctive feature of two-phase flows. In this model the liquid affects the bubble distribution function by three mechanisms: determines the bubble velocities through interfacial stresses, modifies the breakage and coalescence kernels due to turbulence or liquid impurities, and changes the bubble size when a mass flux due to evaporation, condensation or degasification at the gas-liquid interface occurs.

3. THE BUBBLE VOLUME DISTRIBUTION FUNCTION

The above-presented transport equation has enough generality to be applied in many cases. Herein the steady-state spatially-homogeneous problem is numerically solved. It is also assumed that the external forces are negligible and that the only mechanisms of bubble volume change are coalescence and break-up. With this assumptions the total derivative of Equation (6) vanishes. Velocity distributions are not either taken into account, thus leading to the following conservation equation for the bubble volume distribution function $f(V)$:

$$\int_v^\infty b(V,U-V|U)\, f(U)\, dU - \int_0^{v/2} b(U,V-U|V)\, f(V)\, dU + \int_0^{v/2} c(V|U,V-U)\, f(U)\, f(V-U)\, dU - \int_0^\infty c(U+V|U,V)\, f(U)\, f(V)\, dU = 0 \quad (12)$$

For the breakage kernel a critical Weber number criteria is adopted and it is assumed that the bubbles shatter into two bubbles of half their volume. Furthermore, the break-up probability is taken as a constant above the critical volume v_c. Thus,

$$b(V,U-V|U) = 2\, b_0\, \delta(v - u/2)\, \mathcal{H}(u - v_c) \quad (13)$$

where $\mathcal{H}$ is the Heavyside step function.

Two different coalescence kernels are studied. In the first, the coalescence probability is taken to be proportional to the sum of the coalescing bubble volumes:

$$c_1(U+V|U,V) = c_0\,(U + V) \quad (14)$$

A second kernel, often used for rain drops coalescence was studied, which is proportional to the square of the sum of the bubble radii and to the difference between their interfacial area [8]:

$$c_2(U+V|U,V) = c_0'\,(U^{1/3} + V^{1/3})^2\, |U^{2/3} - V^{2/3}| \quad (15)$$

The variables of interest are put in a non-dimensional form taken the critical volume as the reference volume:

$$\hat{V} = V / V_c \quad (16)$$

$$\hat{f}(V) = f(V)\, V_c^2 \quad (17)$$

$$\hat{N}''' = N'''\, V_c \quad (18)$$

$$\hat{\bar{V}} = \alpha / \hat{N}''' \quad (19)$$

And, in order to compare the results obtained with the two kernels, the coalescence probabilities are also non-dimensionalized:

$$\hat{c}_1 = \frac{c_0}{b_0}\left[\hat{U} + \hat{V}\right] \quad (20)$$

$$\hat{c}_2 = \frac{c_0'\, V_c^{1/3}}{b_0}\left[\hat{U}^{1/3} + \hat{V}^{1/3}\right]^2 \left|\hat{U}^{2/3} - \hat{V}^{2/3}\right| \quad (21)$$

And it is required that:

$$\frac{c_0}{b_0} = \frac{c_0'\, V^{1/3}}{b_0} \quad (22)$$

4. NUMERICAL SOLUTION

The integral balance equation (eq. (11)) is numerically solved with an implicit iterative scheme.

The results for the non-dimensional distribution function, computed with the first coalescence kernel (eq. (13)), is shown versus the non-dimensional volume in figure 1. The dimensionless parameter c_o/b_o is put

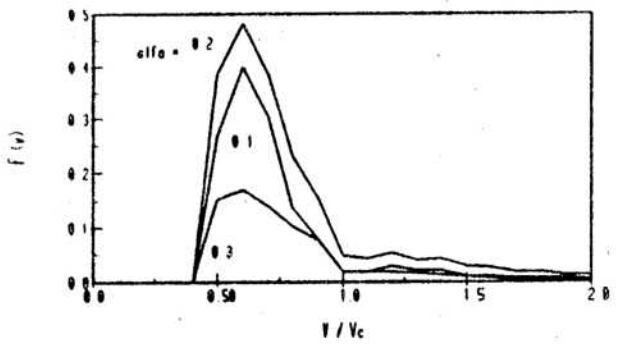

Fig. 1 Bubble volume distribution function versus dimensionless volume for void fractions 0.1, 0.2 and 0.3.

equal to unity. In figure 2 the same is shown in a semilogarithmic plot, in order to see the behavior at large volumes. In both cases it is plotted for three values of the void fraction: 0.1, 0.2 and 0.3. From the figures it can be seen that there are no bubbles with volumes smaller than half the critical volume, which is a direct consequence of the step function that was used for the breakage kernel. It is also seen that the major part of the bubbles have volumes between the critical and half its value. This is an useful feature to estimate the critical volume experimentally. Finally, it should be noted that the number of small bubbles first increases with increasing void fraction, showing that breakage prevails. At higher void fractions the opposite behavior is observed, showing that coalescence prevails (see figure 3).

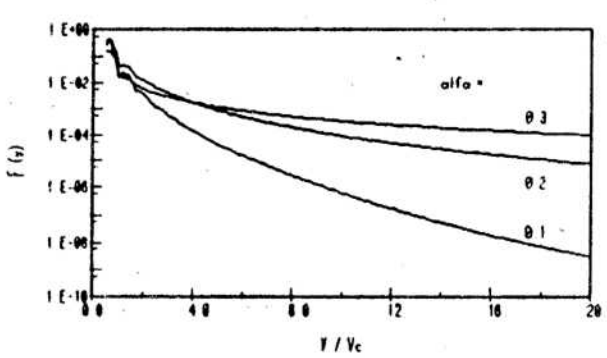

Fig. 2 Idem fig. 1 in a semilogarithmic plot.

Such a behavior is more clearly seen with the aid of figures 4 and 5. In figure 4, curve (a) the dimensionless number density is shown versus the void fraction. It can be seen that above a void fraction of about 0.2 the number of bubbles per unit volume decreases. Consequently, the mean bubble volume shows a slight increase up to a void fraction of 0.2 (see figure 5, curve (a)) and then it increases sharply.

The decrease in the number of bubbles and the increase of their size may be interpreted as the bubbly to slug flow pattern transition. That is, above a certain void fraction the coalescence of small bubbles creates bigger ones, some of them being of an equivalent diameter larger than the tube in which they flow.

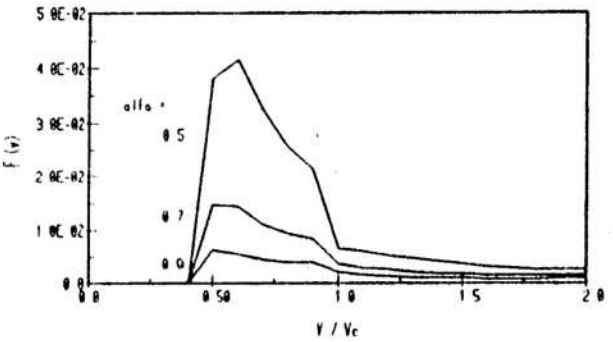

Fig. 3 Idem fig. 1 for void fractions 0.5, 0.7 and 0.9.

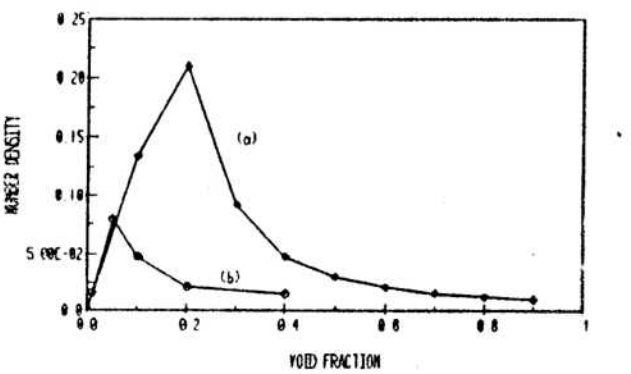

Fig. 4 Number density vs. void fraction computed with coalescence kernel 1 (curve a) and 2 (b).

Finally, the dimensionless interfacial area can be calculated with the known distribution function and it is shown in figure 6 versus the void fraction. It has a similar shape to that of the number density and thus the different flow regimes can be characterized. At low void fractions there exists a region of low to high interfacial area density which corresponds to the bubbly regime, at intermediate values of the void fraction a transition exists in which the interfacial area decreases due to the appearance of larger bubbles which corresponds to the slug flow pattern and at high void fractions the interfacial area density is reduced considerably which corresponds to an annular flow pattern. However, the transitions are not clearly defined by this analysis and should be determined experimentally.

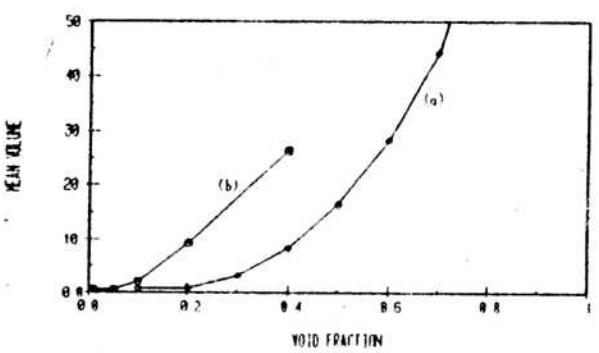

Fig. 5 Increase in mean bubble volume with void fraction for both coalescence kernel.

The same holds for the second coalescence kernel. The behavior is similar to that obtained in the previous case (see figures 4 and 5, curve (b)) with the difference that it is a more coalescing system, thus yielding lower values of the number density and a peak shifted to the left in the N''' vs. α plot.

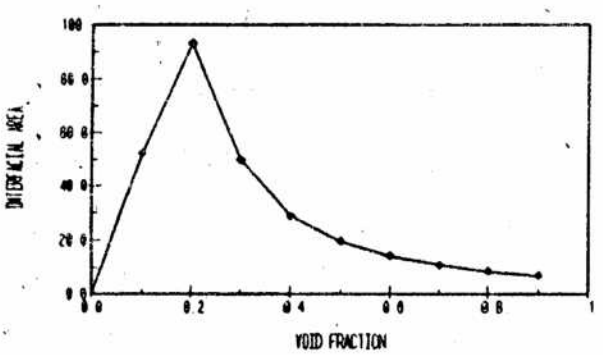

Fig. 6 Interfacial area density vs. void fraction.

An experimental program is being conducted in order to assess the validity of the different kernels and to determine the value of the parameters that appear in them. With the present results it is suggested that the first coalescence kernel be used since it is simpler and the state-of-the-art knowledge does not permit to choose a kernel with other criteria.

5. DISCUSSION

A statistical mechanical description of gas-liquid two-phase flows has been used. The flow was described by means of the bubble volume and velocity distribution function, whose evolution was shown to be governed by a Generalized Boltzmann Transport Equation.

The steady-state spatially-homogeneous case was numerically solved for two different coalescence kernels and a critical Weber number criteria for the break-up probability.

With a coalescence probability proportional to the sum of the bubble volumes it was found that the interfacial area density first increases with void fraction, reaches a maximum and then decreases monotonically. It was interpreted that this behavior corresponds to the different flow regimes that are encountered with increasing void fraction, i.e. bubbly, slug and annular.

With a coalescence kernel proportional to the square of the bubble radii and to the difference between their interfacial area the results were similar but the maximum of the number density appeared at lower values of the void fraction and the number of bubbles was smaller for intermediate and high void fractions.

6. REFERENCES

[1] HULBURT, H.M. and KATZ, S., "Some Problems in Particle Technology. A Statistical Mechanical Formulation", *Chem. Engng. Sci.* 19, pp. 555-574, 1964.

[2] RAMKRISHNA, D., "The Status of Population Balances", *Rev. Chem. Engng.* 3 (1), pp. 49-95, 1985.

[3] HOUNSLOW, M.J., "A Discretized Population Balance for Continuous Systems at Steady State", *AIChE J.* 36 (1), pp. 106-116, 1990.

[4] ACHARD, J.L., "Contribution à l'Etude Théorique des Ecoulements Diphasiques Transitoires", Ph.D. Thesis, \$Grenoble, 1978.

[5] BALESCU, R. "*Equilibrium and Non-equilibrium Statistical Mechanics*", John Wiley & Sons, New York, 1975.

[6] BOFFI, V.C. and SPIGA, G., "Continuity Equation in the Study on Nonlinear Particle-Transport Evolution Problems", *Phys. Rev. A* 29 (2), pp. 782-790, 1984.

[7] ZANETTE, D.H., "Space-Dependent Solutions for the Generalized Boltzmann Equation", *Phys. Fluids A* 2 (2), pp. 274-276, 1990.

[8] WILLIAMS, M.M.R., "Some Topics in Nuclear Aerosol Dynamics", *Prog. Nucl. Energy* 17, pp 1-52, 1986.