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THE RENORMALIZATION OF SINGLE-PARTICLE STATES IN NUCLEAR FIELD THEORY

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Abstract: The single-particle states are renormalized by taking into account the emission and subsequent absorption of a phonon to all orders of perturbation theory. The Ward identity provides a useful normalization condition to the new dressed excitations. Other orthonormalization properties are interpreted in terms of anticommutation and energy weighted sum rules. A two-level model with a monopole phonon is treated in detail as an example.

1. Introduction

The many-fermion problem may be solved, in principle, through the application of the Feynman-Goldstone graphical perturbative expansion [1]). In this procedure, initial and final states are antisymmetrized products of single-particle states. A particularly relevant result of this method is the derivation of collective states by adding up a subset of diagrams, which represents consecutive irreducible interactions between the same pair of fermion lines. The existence of these collective excitations is an essential feature of the nuclear spectrum. The inclusion of these excitations as building blocks is both conceptually and practically advantageous [2]), in spite of the fact that the basic set of states becomes overcomplete. This overcompletness, however, can be systematically eliminated through the application of the nuclear field theory (NFT) rules [3,4]).

In the present paper, we also add up an infinite subset of diagrams, in order to obtain renormalized fermion lines. In principle, this is a generalization of the earlier summation which is implied by the inclusion of Hartree-Fock contributions [1]) in the definition of the single-particle states. In the present case, the added diagrams describe successive interactions of a fermion line with a phonon. Therefore, the fermion lines become "dressed" lines. These dressed fermion lines constitute new building blocks of the nuclear spectrum. The overcompletness implied by this basis can be treated according to the NFT rules [3,4]). Thus, the present paper is a generalization of the NFT (which treates composite particle-hole states in terms of elementary

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bosons) to the case in which the mixture of particle and particle-boson excitations are treated in terms of elementary fermion modes.

Application of the techniques which are developed in the present paper may allow the following:

(i) To simplify the graphical perturbation expansion [3,4]), since the subset of diagrams differing by the successive interaction of a single-fermion line with a boson is reduced to a single diagram.

(ii) To obtain a relation between "bare" and "dressed" single-nuclear states (i.e. between theoretical and experimental energies). This object is similar to the one in ref. [8]). However, both the procedure and the effects which are taken into account are different in the present case.

(iii) To go beyond the lowest order of perturbation theory in treating the influence of the particle-phonon interaction on the properties of single-particle states [see, for instance, the studies on the Pb region [2,5-7])].

2. Procedure

In the propagation of a fermion line, we take into account those processes in which the fermion emits and consecutively absorbs a phonon (fig. 1).

The Dyson equation corresponding to these processes is

$$G(lj, t'-t) = G_{\mathrm{HF}}(j, t'-t)\delta_{jl} - \sum_{pqn} \Lambda^*(jp, n)\Lambda(qp, n) \int \mathrm{d}\tau \mathrm{d}\tau' G_{\mathrm{HF}}(j, \tau - t) \times G_{\mathrm{HF}}(p, \tau'-\tau)G_{\mathrm{B}}(n, \tau'-\tau)G(lq, t'-\tau'), \tag{1}$$

where

$$G(lj, t) = \langle 0|T\{c_l(t)c_j^+(0)\}|0\rangle = \sum_a \langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle \exp(-iE_a t)\theta(t) - \sum_r \langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle \exp(-iE_r t)\theta(-t) \tag{2}$$

is the propagation of a dressed fermion line, and

$$\begin{aligned} G_{\mathrm{HF}}(j, t) &= (1-n_j)\exp(-i\varepsilon_j t)\theta(t) - n_j \exp(-i\varepsilon_j t)\theta(-t), \\ G_{\mathrm{B}}(n, t) &= \exp(-i\omega_n t)\theta(t) + \exp(i\omega_n t)\theta(-t) \end{aligned} \tag{3}$$

are the propagators of a bare fermion (in the Hartree-Fock approximation) and of a phonon, respectively. The particle-phonon vertices are denoted by $\Lambda(jp, n)$,

$$\Lambda^*(pj, n) = \Lambda(jp, n), \tag{4}$$

where the labels (j, p) denote the fermion lines leaving and entering the vertex respectively (see fig. 4).

The intermediate states $|a\rangle$, $|r\rangle$ correspond to systems with one particle added and one particle removed from the closed shell, respectively. In the limit in which these states are of independent-particle type, the "dressed" propagator $G(lj, t)$ tends to the bare propagator $\delta_{jl}G_{\mathrm{HF}}(j, t)$ and the energies E_a, E_r of the intermediate states reduce to the single-particle energies ε_j:

$$E_a = E_a(A+1)-E_0(A) \rightarrow \varepsilon_k, \qquad k \text{ above the Fermi surface,}$$

$$E_r = E_0(A)-E_r(A-1) \rightarrow \varepsilon_i, \qquad i \text{ below the Fermi surface.}$$

A set of diagrams in which an interior fermion line emits and subsequently absorbs a phonon (this process being repeated any number of times) has to be replaced by a single diagram with a dressed fermion line. This line is represented by a straight line with two arrows pointing upwards (downwards) if it corresponds to addition (removal) states. The line is labelled by an initial j and a final l single-particle quantum number

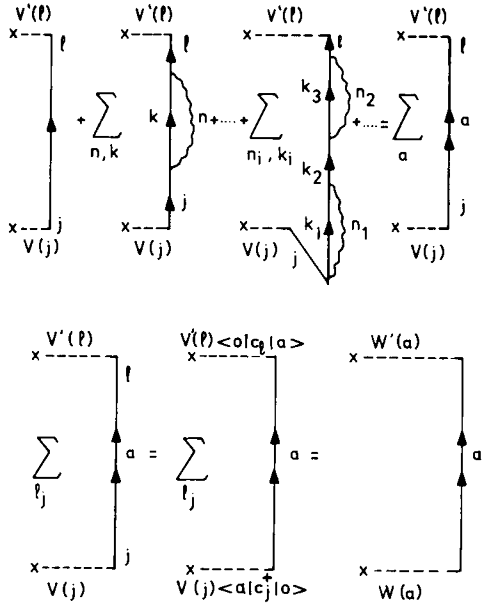

Fig. 1. The propagation of a fermion line, taking into account the successive emission and absorption of a phonon.

(which may correspond either to states above or below the Fermi level). It includes a summation over the number a (r), indicating the intermediate state (see fig. 1).

The line is created at the vertex† $V(j)$ and annihilated at $V'(l)$.

In the calculation of a given diagram one sums over the three quantum numbers j, l, a (r). By performing first the l, j summation, one is left with diagrams where the

† This vertex depends as well on the quantum numbers corresponding to the other lines arriving to the same point. They are omitted since they are not relevant to the following discussion.

dressed fermion is labelled only by the quantum numbers a (r) and is coupled through the vertices:

$$W(a) = \sum_j V(j)\langle a|c_j^+|0\rangle,$$
$$W'(a) = \sum_l V'(l)\langle 0|c_l|a\rangle. \tag{5}$$

Thus it is convenient to replace the (jl) dressed fermion lines by those labelled by a (r) which have a single pole in the propagator at the energy E_a and are coupled through (5).

The use of the fermions a (r) instead of the fermions (l, j) as external lines is consistent with the NFT rule [3]) stating that initial and final states involve the phonons (i.e., the states which are associated with the poles of the particle-hole propagator) and not the particle-hole unperturbed states.

The vertices corresponding to the one-particle creation and annihilation operators are given by the amplitudes $\langle a|c_j^+|0\rangle$ and $\langle r|c_j|0\rangle$, respectively. The state j may be either above or below the Fermi level. The processes implied in these amplitudes are discussed in connection with the examples treated in sect. 3 (see figs. 6 and 7).

We proceed now to calculate the energies E_a, E_r and the amplitudes $\langle a|c_j^+|0\rangle$, $\langle r|c_j|0\rangle$. The usual time integrations and Fourier transforms yield the equation

$$\begin{aligned}
\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle}{k-E_a} &= \delta_{jl}' \frac{1-n_j}{k-\varepsilon_j} + \sum_{pqn} \Lambda^*(jp,n)\Lambda(qp,n) \\
&\times \Bigg((1-n_p)\Bigg[\frac{1-n_j}{(\varepsilon_{jp}-\omega_n)(k-\varepsilon_j)}\left(\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{\varepsilon_j - E_a} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{\varepsilon_j - E_r}\right) \\
&+ \frac{1}{(\varepsilon_{jp}-\omega_n)(k-\varepsilon_p-\omega_n)}\left(\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{E_a-\varepsilon_p-\omega_n} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{E_r-\varepsilon_p-\omega_n}\right) \\
&+ \sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{(k-E_a)(E_a-\varepsilon_j)(E_a-\varepsilon_p-\omega_n)}\Bigg] \\
&+ n_p\Bigg[\frac{1-n_j}{(\varepsilon_{jp}+\omega_n)(k-\varepsilon_j)}\left(\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{\varepsilon_j - E_a} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{\varepsilon_j - E_r}\right) \\
&+ \sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{(k-E_a)(E_a-\varepsilon_j)(E_a+\omega_n-\varepsilon_p)}\Bigg]\Bigg),
\end{aligned} \tag{6}$$

where $\varepsilon_{jp} = \varepsilon_j - \varepsilon_p$, and a similar expression for the removal part.

At the poles $k = E_a$, E_r, the following homogeneous equations have to be satisfied:

$$(E_a-\varepsilon_j)\langle a|c_j^+|0\rangle = \sum_{pqn} \Lambda^*(jp,n)\Lambda(qp,n)\langle a|c_q^+|0\rangle\left(\frac{1-n_p}{E_a-\varepsilon_p-\omega_n} + \frac{n_p}{E_a-\varepsilon_p+\omega_n}\right),$$

$$(E_r-\varepsilon_j)\langle 0|c_j^+|r\rangle = \sum_{pqn} \Lambda^*(jp,n)\Lambda(qp,n)\langle 0|c_q^+|r\rangle\left(\frac{1-n_p}{E_r-\varepsilon_p-\omega_n} + \frac{n_p}{E_r-\varepsilon_p+\omega_n}\right). \tag{7}$$

Hence the energies E_a, E_r are given by the roots of the determinantal equation

$$0 = \det\left((\varepsilon_j - E)\delta_{jq} + \sum_{pn} \Lambda^*(jp, n)\Lambda(qp, n)\left(\frac{1-n_p}{E-\varepsilon_p-\omega_n} + \frac{n_p}{E-\varepsilon_p+\omega_n}\right)\right). \tag{8}$$

Thus, apparently, both the addition and removal states have the same energy. This statement is obviously wrong. Eq. (8) can be reconciled with the correct results by noting that for a given pole E, either $\langle a(E)|c_j^+|0\rangle$ or $\langle r(E)|c_j|0\rangle$ vanish for all j. A convenient procedure which may be used to distinguish between addition and removal roots is the prescription given by Martin and Schwinger [9]). Through a suitable definition of the zero of the single-particle energy, the addition poles E_α are positive, while the removal poles E_r are negative.

At the poles $k = \varepsilon_j$, (6) yields

$$\delta_{jl} = \sum_{pqn} \Lambda^*(jp, n)\Lambda(qp, n)\left(\frac{1-n_p}{\varepsilon_{pj}+\omega_n} + \frac{n_p}{\varepsilon_{pj}-\omega_n}\right) \times\left(\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{\varepsilon_j - E_a} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{\varepsilon_j - E_r}\right). \tag{9}$$

Finally, at the poles $k = \varepsilon_p + \omega_n$ and $k = \varepsilon_p - \omega_n$ one obtains, respectively,

$$0 = (1-n_p)\sum_q \frac{\Lambda^*(jp, n)\Lambda(qp, n)}{\varepsilon_{pj}+\omega_n}\left[\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{E_a-\varepsilon_p-\omega_n} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{E_r-\varepsilon_p-\omega_n}\right],$$
$$0 = n_p\sum_q \frac{\Lambda^*(jp, n)\Lambda(qp, n)}{\varepsilon_{pj}-\omega_n}\left[\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{E_a-\varepsilon_p+\omega_n} + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{E_r-\varepsilon_p+\omega_n}\right]. \tag{10}$$

Eqs. (9) and (10) may be elaborated as follows: adding the last two equations and summing over p and n one obtains

$$0 = \sum_{pqn} \Lambda^*(jp, n)\Lambda(qp, n)\left(\sum_a \langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle\left[\frac{1-n_p}{(\varepsilon_{pj}+\omega_n)(E_a-\varepsilon_p-\omega_n)} + \frac{n_p}{(\varepsilon_{pj}-\omega_n)(E_a-\varepsilon_p+\omega_n)}\right] + \sum_r \langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle\left[\frac{1-n_p}{(\varepsilon_{pj}+\omega_n)(E_r-\varepsilon_p-\omega_n)} + \frac{n_p}{(\varepsilon_{pj}-\omega_n)(E_r-\varepsilon_p+\omega_n)}\right]\right). \tag{11}$$

The summation of eqs. (9) and (11) and the application of eqs. (7) yield

$$\delta_{jl} = \sum_{pqn} \Lambda^*(jp, n)\Lambda(qp, n)\left(\sum_a \frac{\langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle}{\varepsilon_j-E_a}\left(\frac{1-n_p}{\varepsilon_p-E_a+\omega_n} + \frac{n_p}{\varepsilon_p-E_r-\omega_n}\right) + \sum_r \frac{\langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle}{\varepsilon_j-E_r}\left(\frac{1-n_p}{\varepsilon_p-E_r+\omega_n} + \frac{n_p}{\varepsilon_p-E_r-\omega_n}\right)\right)$$
$$= \sum_a \langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle + \sum_r \langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle. \tag{12}$$

Therefore, one obtains the familiar one-particle sum rule

$$\langle 0|\{c_l, c_j^+\}|0\rangle = \delta_{l,j}. \tag{13}$$

One obtains another familiar sum rule [8]) by adding the two eqs. (10) and using again eq. (7),

$$0 = \sum_{pqn} \Lambda^*(jp, n)\Lambda(qp, n)\left[\sum_a \langle 0|c_l|a\rangle\langle a|c_q^+|0\rangle\left(\frac{1-n_p}{E_a-\varepsilon_p-\omega_n}+\frac{n_p}{E_a-\varepsilon_p+\omega_n}\right)\right.$$
$$\left.+\sum_r \langle 0|c_q^+|r\rangle\langle r|c_l|0\rangle\left(\frac{1-n_p}{E_r-\varepsilon_p-\omega_n}+\frac{n_p}{E_r-\varepsilon_p+\omega_n}\right)\right]$$
$$= \sum_a \langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle(E_a-\varepsilon_j)+\sum_r \langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle(E_r-\varepsilon_j).$$

This condition is of the form

$$\varepsilon_j\delta_{jl} = \sum_a E_a\langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle+\sum_r E_r\langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle, \tag{14}$$

which expresses the conservation of the energy weighted sum rule for the one-body transfer processes. In ref. [8]), eq. (14) was predicted to hold for the present phonon-particle coupling model in which no four-point vertices are present in the bare fermion line.

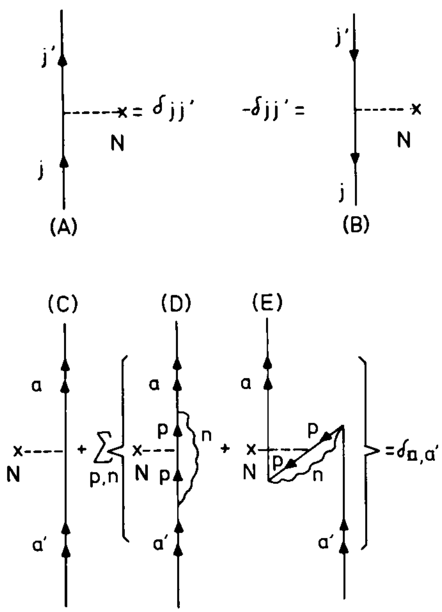

Fig. 2. The vertices of the number operator and the graphical representation of the matrix element of this operator between dressed states.

The normalization conditions (12) imply a summation over all the roots a, r and, therefore, they are cumbersome to use since usually one is interested in a few states.

Alternatively, a normalization condition is obtained by requiring the states $|a\rangle$ and $|r\rangle$ to be eigenfunctions of the opeiator corresponding to the number of particles, with eigenvalues 1 and -1, respectively. This is contained in the Ward [1]) or Ward-Pitaerskii [1]) identity † which states that the electron self-energy and vertex corrections mutually cancel in the limit of low momentum transfer. The limit occurs in a spherical shell model system for a monopole operator with constant matrix elements (i.e., the number of particle operator).

The bare vertices of the number operator are given in figs. 2a and b. The sum of the three graphs of figs. 2c–e implies the orthonormalization condition:

$$\delta_{a,a'} = \sum_j \langle a|c_j^+|0\rangle\langle 0|c_j|a'\rangle + \sum_{pn} \frac{(\sum_j \Lambda^*(pj,n)\langle a|c_j^+|0\rangle)(\sum_j \Lambda(pj,n)\langle 0|c_j|a'\rangle)}{(E_a-\varepsilon_p-\omega_n)(E_{a'}-\varepsilon_p-\omega_n)}(1-n_p)$$
$$+\sum_{pn} \frac{(\sum_j \Lambda(jp,n)\langle a|c_j^+|0\rangle)(\sum_j \Lambda^*(jp,n)\langle 0|c_j|a'\rangle)}{(E_a-\varepsilon_p+\omega_n)(E_{a'}-\varepsilon_p+\omega_n)} n_p, \tag{15a}$$

$$\delta_{r,r'} = \sum_j \langle r|c_j|0\rangle\langle 0|c_j^+|r'\rangle + \sum_{pn} \frac{(\sum_j \Lambda^*(jp,n)\langle r|c_j|0\rangle)(\sum_j \Lambda(jp,n)\langle 0|c_j^+|r'\rangle)}{(E_r-\varepsilon_p+\omega_n)(E_{r'}-\varepsilon_p+\omega_n)} n_p$$
$$+\sum_{pn}(1-n_p)\frac{(\sum_j \Lambda(pj,n)\langle r|c_j|0\rangle)(\sum_j \Lambda^*(pj,n)\langle 0|c_j^+|r'\rangle)}{(E_r-\varepsilon_p-\omega_n)(E_{r'}-\varepsilon_p-\omega_n)}. \tag{15b}$$

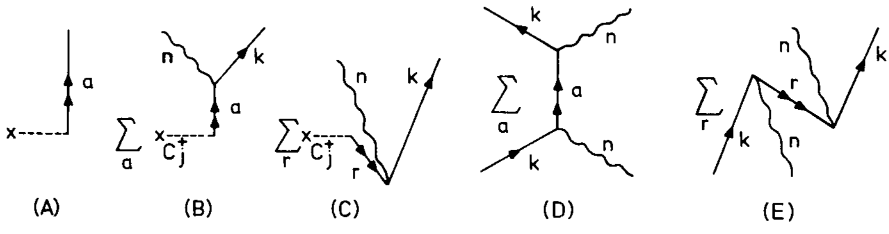

Fig. 3. The particle-phonon spurious states. Diagram (A) is the correct one representing the population of the final state via the one-body stripping. Diagrams (B) and (C) correspond to the population of the spurious state and their sum cancels as the sum of the energy diagrams (D) and (E).

The present procedure mixes a bare fermion with a bare fermion plus boson state, but it is not symmetrical in these unperturbed states. We may replace any internal bare fermion line by a dressed fermion line (and we thus take into account a whole subset of higher order graphs). Those graphs in which the particle and the phonon

† The application of this identity to the nuclear spectrum has been recently discussed by Paar [10]).

start and end at the same point (without interacting with the remaining lines) have to be disregarded. This is analogous to the rule of the NFT eliminating diagrams containing bubbles.

In the case of external lines, not only the bare fermion lines but also the (bare particle + phonon) lines disappear (if both the particle and the phonon join the remaining part of the diagram at the same point). This is again analogous to another NFT rule, which eliminates particle-hole pairs that may be replaced by a phonon from the initial or final states. For instance, both diagrams 3a and 3b + 3c represent essentially the same process if the root a in fig. 3a represents a predominantly particle-phonon state. The correct diagram to use is 3a. Diagrams 3b and 3c have to be disregarded because of the rule mentioned above. However, it is interesting to note that its contribution vanishes,

$$\langle k, n|c_j^+|0\rangle = \sum_{a,l} \langle 0|c_l|a\rangle\langle a|c_j^+|0\rangle \frac{\Lambda^*(lk, n)}{\varepsilon_k+\omega_n-E_a} + \sum_{r,l} \langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle \frac{\Lambda^*(lk, n)}{\varepsilon_k+\omega_n-E_r} = 0, \tag{16}$$

according to eq. (10).

Within the present formalism, the particle-phonon state is spurious and thus improper as initial or final state. As befits a spurious state, its energy is the unperturbed energy and its amplitude vanishes [cf. ref. [11])]. The first part of the last statement is easily verified, since the only possible diagrammatic contributions are given in figs. 3d and 3e:

$$E = \varepsilon_k+\omega_n+ \sum_{j,l} \Lambda^*(jk, n)\Lambda(lk, n)\left[\sum_a \frac{\langle 0|c_j|a\rangle\langle a|c_l^+|0\rangle}{\varepsilon_k+\omega_n-E_a} + \sum_r \frac{\langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle}{\varepsilon_k+\omega_n-E_r}\right] = \varepsilon_k+\omega_n \tag{17}$$

[cf. again eq. (10)].

The square of the amplitude of the unperturbed state in the final state is given by the derivative of the final energy E with respect to the unperturbed energy [12]):

$$X^2(k, n) = \frac{\mathrm{d}E}{\mathrm{d}(\varepsilon_k+\omega_n)} = 0, \tag{18}$$

which yields the normalization condition

$$1 = \sum_{jl} \Lambda^*(jk, n)\Lambda(lk, n)\left[\sum_a \frac{\langle 0|c_j|a\rangle\langle a|c_l^+|0\rangle}{(\varepsilon_k+\omega_n-E_a)^2} + \sum_r \frac{\langle 0|c_j^+|r\rangle\langle r|c_l|0\rangle}{(\varepsilon_k+\omega_n-E_r)^2}\right]. \tag{19}$$

Eq. (9) expresses the normalization condition for fixed particle states (j, j'); the Ward identity yields the normalization condition for a particular dressed state $(|a\rangle, |r\rangle)$. Finally, eq. (19) is a normalization condition corresponding to a fixed particle-phonon state $|k, n\rangle$.

3. Application to the monopole model

We assume that only one monopole phonon is present and that particles move in two levels which are labelled by the quantum number σ ($\sigma = 1$ or $\sigma = -1 = \bar{1}$). The two levels have the same degeneracy 2Ω and they are separated by the distance ε ($\varepsilon_\sigma = \frac{1}{2}\sigma\varepsilon$). Within each level, the single-particle states are characterized by the quantum number m. The particle-phonon vertices are given in fig. 4.

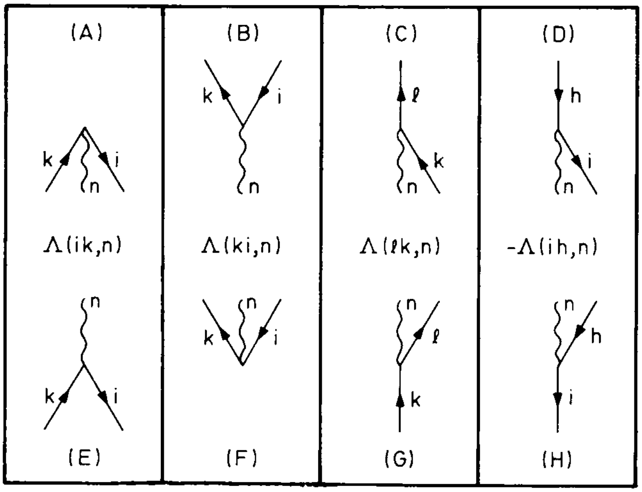

Fig. 4. The particle-phonon vertices. In the case of the monopole interaction the vertices (a) and (f) have the value $-\Lambda_b$, the vertices (b) and (e), $-\Lambda$; and the remaining vertices, $-\Lambda_s$. In this figure the labels k, l (h, i) denote states above (below) the Fermi level.

In the present case, there is a determinantal equation (8) for each m. Since this equation is the same for all values of m, we drop this subindex. We obtain

$$0 = \begin{vmatrix} \frac{1}{2}\varepsilon - E + \dfrac{\Lambda_s^2}{E-\omega-\frac{1}{2}\varepsilon} + \dfrac{\Lambda_b^2}{E+\omega+\frac{1}{2}\varepsilon} & \dfrac{2\Lambda_b\Lambda_s(\frac{1}{2}\varepsilon+\omega)}{E^2-(\frac{1}{2}\varepsilon+\omega)^2} \\ \dfrac{2\Lambda_b\Lambda_s(\frac{1}{2}\varepsilon+\omega)}{E^2-(\frac{1}{2}\varepsilon+\omega)^2} & -\frac{1}{2}\varepsilon - E + \dfrac{\Lambda_b^2}{E-\omega-\frac{1}{2}\varepsilon} + \dfrac{\Lambda_s^2}{E+\omega+\frac{1}{2}\varepsilon} \end{vmatrix} \tag{20}$$

We first consider the case in which there are no scattering vertices ($\Lambda_s = 0$). The determinantal equation separates into two equations, corresponding to the upper and lower single-particle levels, respectively:

$$\begin{aligned} 1 &= \Lambda_b^2/(E+\omega+\tfrac{1}{2}\varepsilon)(E-\tfrac{1}{2}\varepsilon) \equiv F_1(E), \\ 1 &= \Lambda_b^2/(E-\omega-\tfrac{1}{2}\varepsilon)(E+\tfrac{1}{2}\varepsilon) \equiv F_{\bar{1}}(E). \end{aligned} \tag{21}$$

The functions $F_\sigma(E)$ are plotted in fig. 5 as a function of E. The intersections with the horizontal line at $y = 1$ determine the eigenvalues $E_a(\sigma)$ and $E_r(\sigma)$. Each of the

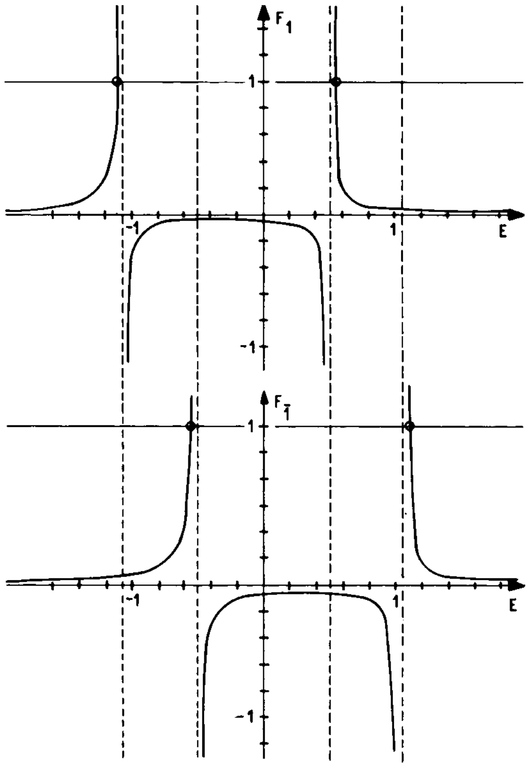

Fig. 5. Graphical representation of the dispersion equations (21). The parameters of the drawing are $\Omega = 5$, $\Lambda_s = 0$, $\Lambda_b^2 = \Lambda^2 = x^2/8\Omega\omega$, $\omega = \sqrt{1-x}$, $x = 0.7$ and $\varepsilon = 1$. The intersections determining the roots are encircled.

two equations has a positive and a negative root, corresponding to the addition and to the removal of a particle, respectively. These roots are

$$\begin{aligned} E_a(1) &= -E_r(\bar{1}) = -\tfrac{1}{2}(\omega-\Delta), \\ E_r(1) &= -E_a(\bar{1}) = -\tfrac{1}{2}(\omega+\Delta), \end{aligned} \tag{22}$$

where

$$\Delta = [(\varepsilon+\omega)^2+4\Lambda_b^2]^{\frac{1}{2}}. \tag{23}$$

Since there is only one addition (removal) solution for each of the eqs. (21), the addition (removal) states $|a_\sigma\rangle$ ($|r_\sigma\rangle$) can be distinguished by the label σ.

The existence of only diagonal terms in the determinantal equation insures the vanishing of the cross amplitudes

$$\langle a_\sigma|c_{\bar{\sigma}}^{+}|0\rangle = \langle r_\sigma|c_{\bar{\sigma}}|0\rangle = 0. \tag{24}$$

The absolute value † of the residues are determined through the normalization conditions (13) and (15). These equations are

$$\begin{aligned} 1 &= |\langle a_\sigma | c_\sigma^+ |0\rangle|^2 + |\langle r_\sigma | c_\sigma |0\rangle|^2, \\ \tfrac{1}{2}\sigma\varepsilon &= E_a(\sigma)|\langle a_\sigma | c_\sigma^+ |0\rangle|^2 + E_r(\sigma)|\langle r_\sigma | c_\sigma |0\rangle|^2, \end{aligned} \tag{25}$$

which yield the values

$$\begin{aligned} |\langle a_1 | c_1^+ |0\rangle|^2 &= |\langle r_{\bar{1}} | c_{\bar{1}} |0\rangle|^2 = \frac{1}{2}\left(1 + \frac{\varepsilon+\omega}{\Delta}\right), \\ |\langle r_1 | c_1 |0\rangle|^2 &= |\langle a_{\bar{1}} | c_{\bar{1}}^+ |0\rangle|^2 = \frac{1}{2}\left(1 - \frac{\varepsilon+\omega}{\Delta}\right). \end{aligned} \tag{26}$$

The inclusion of backwards correlations is made through the coupling of the creation of a particle state with energy $\frac{1}{2}\varepsilon$ and the annihilation of a hole-phonon state with (negative) energy $-\frac{1}{2}\varepsilon - \omega$ (alternatively a hole state of energy $\frac{1}{2}\varepsilon + \omega$). This is similar to what happens in the usual RPA solution for particle-hole and

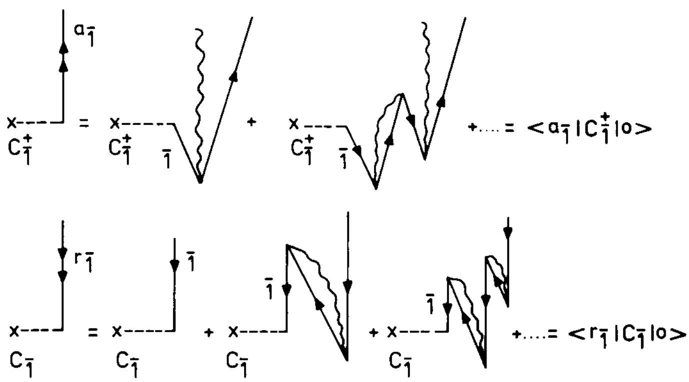

Fig. 6. The amplitudes for the creation and annihilation of one particle in a state below the Fermi level and the corresponding series of diagrams using bare fermion lines.

hole-particle amplitudes. It can be interpreted in terms of a summation of an infinite number of diagrams. As a consequence of this coupling, by acting with the operator $c_{\bar{1}}^+$ on the ground state of the closed shell system one can create a particle in the level below and thus populate the particle-phonon state (fig. 6). The destruction of a particle is also renormalized by backward correlations (see fig. 6). These two processes are measured by the spectroscopic factors (26).

† In the present case, each state $|a_\sigma\rangle$, $|r_\sigma\rangle$ is connected through a single matrix element of the one-body transfer operators with the closed-shell system $|0\rangle$. Consequently, the phase of these non-vanishing matrix elements cannot be determined.

We note that in the present fermion case, the sum of the square of the amplitudes is one [eq. (12)]. This is unlike the RPA case in which the squares of the backwards amplitudes have to be subtracted in the normalization procedure.

We have used the normalization conditions corresponding to the fermion and the energy weighted sum rules in order to determine the square of the amplitudes. It is instructive to verify that the normalization conditions (15) and (19) are also satisfied, as they should.

If $a = a' = a_1$ (a_1 = lowest addition root), then $j = 1$ and the intermediate state $p = \bar{1}$ in eq. (15a) (since no scattering vertices are allowed). In such case, only diagrams 2c and 2e contribute to the matrix element of the number operator, namely

$$\langle a_1|N|a_1\rangle = \langle a_1|c_1^+|0\rangle^2[1+\Lambda_{\mathrm{b}}^2/(E_{a_1}-\varepsilon_{\bar{1}}+\omega)^2]. \tag{27}$$

The factor within brackets may be elaborated by using the dispersion eq. (21) and the expression for the root E_a given in eq. (22):

$$1+\frac{\Lambda_{\mathrm{b}}^2}{(E_{a_1}-\varepsilon_{\bar{1}}+\omega)^2} = 1+\frac{E_{a_1}-\varepsilon_1}{E_{a_1}-\varepsilon_{\bar{1}}+\omega} = \frac{2\Delta}{\Delta+\varepsilon+\omega}, \tag{28}$$

which is the inverse of the square of the amplitude $|\langle a_1|c_1^+|0\rangle|^2$ according to (26). Thus, the Ward identity is satisfied.

If $a = a' = a_2$, then the intermediate state p is a particle and only diagrams 2a contribute and a similar verification is obtained.

The condition (19) reads

$$\begin{aligned} 1 &= \left[\sum_a \frac{\langle 0|c_{\bar{1}}|a\rangle\langle a|c_{\bar{1}}^+|0\rangle}{(\varepsilon_1+\omega-E_a)^2}+\sum_r \frac{\langle 0|c_{\bar{1}}^+|r\rangle\langle r|c_{\bar{1}}|0\rangle}{(\varepsilon_1+\omega-E_r)^2}\right]\Lambda_{\mathrm{b}}^2 \\ &= \Lambda_{\mathrm{b}}^2\left(\frac{\langle 0|c_{\bar{1}}|a_{\bar{1}}\rangle\langle a_{\bar{1}}|c_{\bar{1}}^+|0\rangle}{(\varepsilon_1+\omega-E_a(\bar{1}))^2}+\frac{\langle 0|c_{\bar{1}}^+|r_{\bar{1}}\rangle\langle r_{\bar{1}}|c_{\bar{1}}|0\rangle}{(\varepsilon_1+\omega-E_r(\bar{1}))^2}\right). \end{aligned} \tag{29}$$

The case with only scattering vertices present ($\Lambda = \Lambda_{\mathrm{b}} = 0$) can also be easily treated, since the determinant again becomes diagonal. In this case, the single-particle is mixed with the particle-phonon state.

If both backwards and scattering vertices are present, the determinantal equation is quadratic in E^2, the roots being

$$\begin{aligned} E^2 &= \tfrac{1}{2}[\tfrac{1}{4}\varepsilon^2+(\tfrac{1}{2}\varepsilon+\omega)^2+2(\Lambda_{\mathrm{s}}^2+\Lambda_{\mathrm{b}}^2)]\pm\tfrac{1}{2}\eta, \\ \eta^2 &= \omega^2(\varepsilon+\omega)^2+4\Lambda_{\mathrm{s}}^2(\varepsilon+\omega)^2+4\Lambda_{\mathrm{b}}^2\omega^2. \end{aligned} \tag{30}$$

There are again four roots, the two positive ones corresponding to addition states and the two negative ones, to removal states. These energies are

$$E_a(1) = -E_r(1) = \tfrac{1}{2}\varepsilon + \left[\frac{\Lambda_b^2}{\varepsilon+\omega} - \frac{\Lambda_s^2}{\omega}\right] + O(\Lambda^4),$$
$$E_a(2) = -E_r(2) = \tfrac{1}{2}\varepsilon + \omega + \left[\frac{\Lambda_b^2}{\varepsilon+\omega} + \frac{\Lambda_s^2}{\omega}\right] + O(\Lambda^4). \tag{31}$$

Eqs. (7) yield the ratio between the amplitudes

$$\langle a_i|c_\sigma^+|0\rangle/\langle a_i|c_{\bar\sigma}^+|0\rangle \text{ and } \langle 0|c_\sigma^+|r_i\rangle/\langle 0|c_{\bar\sigma}^+|r_i\rangle.$$

Using the ratios, plus the normalization conditions (12) and (14), one obtains the amplitudes

$$\langle a_1|c_1^+|0\rangle = \langle r_1|c_{\bar 1}|0\rangle = 1 - \tfrac{1}{2}\left[\frac{\Lambda_s}{\omega^2} + \frac{\Lambda_b^2}{(\varepsilon+\omega)^2}\right] + O(\Lambda^4),$$
$$\langle a_2|c_1^+|0\rangle = \langle r_2|c_{\bar 1}|0\rangle = \Lambda_s/\omega + O(\Lambda^3),$$
$$\langle a_1|c_{\bar 1}^+|0\rangle = \langle r_1|c_1|0\rangle = -\Lambda_b\Lambda_s\frac{\varepsilon+2\omega}{\varepsilon\omega(\varepsilon+\omega)} + O(\Lambda^4),$$
$$\langle a_2|c_{\bar 1}^+|0\rangle = \langle r_2|c_1|0\rangle = \Lambda_b/(\varepsilon+\omega) + O(\Lambda^3), \tag{32}$$

where the phases in the first two lines have been arbitrarily chosen.

4. The particle-phonon interaction in NFT

The present techniques associated with the treatment of the particle-phonon interaction are independent of the origin of the phonon. However, if the phonons

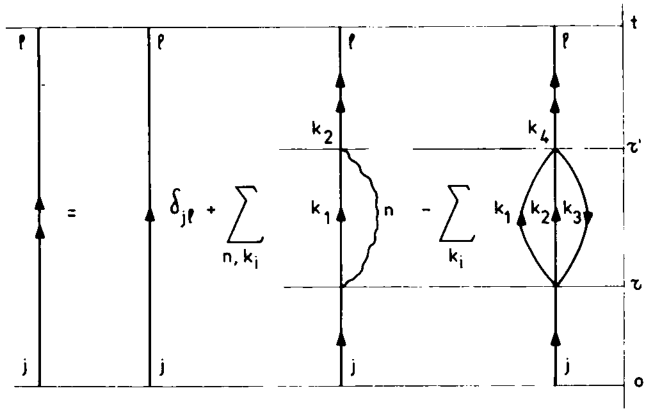

Fig. 7. Corrections to eq. (1) when the exchange terms of the two-body interaction are included in the definition of the boson.

are only formally independent from the fermion degree of freedom (as in NFT), and if both direct and exchange components of the two-body interaction have been used in the defmition of the boson, the kernel (1) has to be supplemented with a (negative) contribution representing the propagation of a free two-particle one-hole triplet between the times τ and τ' [see ref. [3]] as shown in fig. 7.

5. Conclusions

The processes in which a fermion consecutively emits and absorbs a phonon may be easily taken into account in all orders of perturbation theory. The resultant energies are obtained by solving the eigenvalue eq. (8). The amplitudes of the bare single-particle states in the dressed states are used to renormalize the vertices. Their relative values are obtained from the homogeneous eq. (7); their absolute value may be derived from the normalization condition (9), which, together with eqs. (10) imply the validity of the anticommutation and energy weighted sum rules. An alternative (and probably more useful) method to normalize the states is obtained through application of the Ward identity.

As an example, the simple but meaningful case of particles moving in two levels and interacting with a monopole phonon has been treated in detail.

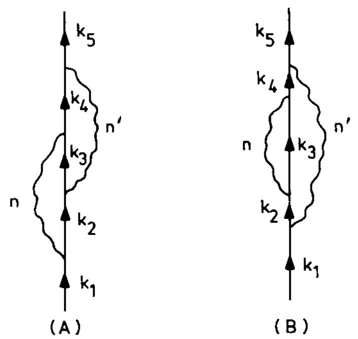

Fig. 8. The Ω^{-2} processes due to the particle-phonon interaction.

In the present paper, only the simplest kernel involving the particle-phonon interaction has been considered. More complicated processes, such as those shown in fig. 8, may be taken into account by including the corresponding kernels in the Dyson equation (1). The techniques described in the present paper can be applied as well. However, even if only the kernel (1) is included, it is still advantageous to use the renormalized fermions (1), in spite of the fact that the lowest order graphs associated with the kernels of fig. 8 are of the same order as those arising from the second iteration in eq. (1)

The use of the Ward-Pitaerskii identity for normalizing dressed states can also be applied in a more general framework than the one dealt with in this paper.

The problems arising from the overcompletness of the basis was found to have the same solution as in the ordinary NFT [11]). Spurious states have the same energy as the unperturbed ones and cannot be populated in any physical process.

In the application of the NFT, the expansion parameter is taken to be Ω^{-1}, where Ω is the number of single-particle states which effectively contribute to the collective phonons. Calculations to first order in this parameter have been succesful in the Pb region. Around other doubly magic nuclei, however, the j-subshells have a smaller degeneracy, and an expansion to order Ω^{-2} may be required. A straightforward application of the present summation eliminates a large number of the corresponding diagrams, which otherwise becomes prohibitively too large. Inclusion of the process of fig. 8 in the definition of the dressed fermions would only yield contributions of order Ω^{-3}.

The other main advantage of the use of the renormalized fermions (1) is to avoid the shortcomings inherent to the lowest order of the Rayleigh-Schrödinger perturbation theory.

Discussions with Dr. R. J. Liotta have stimulated this work.

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