

Longitudinal Magnetomorphic Effect in Indium Films*

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Measurements of magnetomorphic effects in indium films of thickness between 0.003 and 0.01 cm have been made at 4.2 and 1.5°K. The results were compared with theoretical calculations based on the free-electron model. To make these comparisons, it was necessary to discount the bulk magnetoresistance by means of a modified Kohler's rule suggested by Olsen. From the comparison of the data with the theory, it was possible to deduce the values of the electronic momentum mv and the product of the bulk resistivity with the bulk mean free path $\rho_b l$. Data previously obtained by Olsen in wires have been analyzed with the use of measurements of Kohler's rule and of a criterion for the comparison different from the one used by Olsen. The experimental value of $\rho_b l$ is discussed on the basis of an analysis made by Bate *et al.*, in which the anisotropy of the mean free path is taken into account.

INTRODUCTION

THE effect of the electron scattering at the boundary of a thin conductor can be studied by measuring the size dependence of the electrical resistivity at low temperature in sufficiently pure materials. Under these conditions, the electronic mean free path is comparable to the thickness of the sample, and thus the effect is easy to observe. Experiments in thin films and thin wires have been made by several investigators,¹ and theoretical calculations for these cases have been carried out by Fuchs² and Dingle.³ In both cases use is made of the free-electron model, and the Boltzmann equation is solved with suitable boundary conditions and the assumption of a unique and isotropic relaxation time. By comparing the experimental results with theory, it is, in principle, possible to determine the values of the bulk resistivity ρ_b and the bulk mean free path l of the sample. However, for this to be possible in practice, we must assume that ρ_b and l are each the same in all the samples. Experimentally, this is a very difficult condition to achieve, since the different samples, even when cut from the same material, may contain different amounts of impurities as a consequence of the preparation process.

According to theoretical calculations, a more powerful way to determine ρ_b , l , and also the electronic momentum mv , is by measuring the electrical resistivity size effect in the presence of a magnetic field. An important advantage of measuring magnetomorphic effects and comparing the result with theory is the possibility of obtaining the values of ρ_b , l , and mv from a single sample.

Because of the many possibilities of geometry and field orientations with respect to the sample and

current, several effects can be studied. We will be interested in the case of thin films in longitudinal magnetic fields. This problem has been studied theoretically by Yi-Han Kao,⁴ using the free-electron model and Chambers's kinetic method.⁵ For the case of wires, the theory predicts that the magnetoresistance decreases monotonically as the magnetic field is increased; for thin films, a maximum appears in the low-field region.

Magnetoacoustic measurements in indium⁶ show that the free-electron model describes the features of the Fermi surface quite well. However, measurements of longitudinal magnetomorphic effects in thin indium wires⁷ show no reasonable agreement with the theory. On the other hand, aluminum, which has a Fermi surface quite similar to that of indium, showed good agreement between theory and experiment,⁸ for the longitudinal magnetomorphic effects.

Metals with a nonspherical Fermi surface show intrinsic magnetoresistance, which must be discounted in order to compare theory and experiment. The value measured is the superposition of both size effect and bulk magnetoresistance, and it is not possible to separate them in an easy way.

Unfortunately, there is no theoretical treatment available to tell how each effect contributes to the total magnetoresistance. Olsen, however, has been able

TABLE I. Dimensions and bulk resistivity ρ_b of the samples used in the determination of Kohler's rule for indium.

Sample	Thickness (cm)	ρ_b ($10^{-10}\Omega$ cm)
I	0.2	43
II	0.2	179
III	0.2	68
IV	0.2	68

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¹ For references see J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960), p. 468.

² K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).

³ R. B. Dingle, Proc. Roy. Soc. (London) **A201**, 545 (1950).

⁴ Yi-Han Kao, Phys. Rev. **138**, 1412 (1965).

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⁶ J. A. Rayne and B. S. Chandrasekhar, Phys. Rev. **125**, 1952 (1962).

⁷ J. L. Olsen, Helv. Phys. Acta, **31**, 713 (1958).

⁸ O. S. Lutes and D. A. Clayton, Phys. Rev. **138**, A1448 (1965).

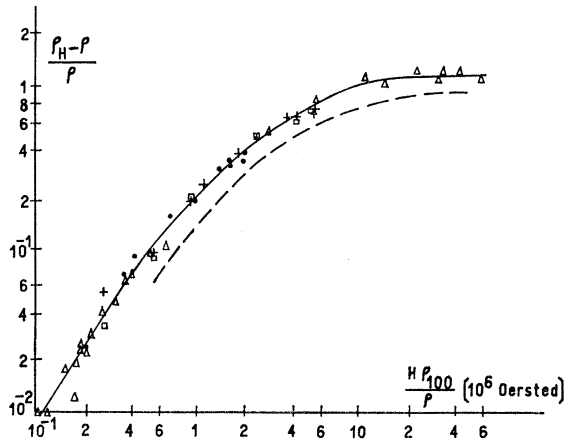


FIG. 1. Determination of Kohler's function. ρ_H is the resistivity determined in field H applied longitudinally; ρ is the zero-field resistivity; ρ_{100} is the resistivity at 100°K. Δ , Sample I; \bullet , sample II; $+$, sample III; \square , sample IV. The dashed curve gives Kohler's function from data obtained by Olsen.

to separate the two effects by means of a suitable modification of Kohler's rule.⁷

We have performed measurements in thin films to see whether it is possible to give values to l , ρ_b and mv in order to match the experimental and theoretical curves.

EXPERIMENTAL DETAILS

Sample Preparation

Seven samples—I, V, VI, VII, VIII, IX, and X—were cut from an ingot of 99.9999% pure indium from Consolidated Mining and Smelting Company of Canada, Ltd.; the material was rolled between two Teflon plates from an initial thickness of 0.3 cm to the final one. During the rolling process the surface of the samples and the Teflon plates were thoroughly washed with acetone. Samples II and III were rolled with a similar process, from a 2-cm-diam cylinder of material of lower purity. The dimensions of all the samples are shown in Tables I and II.

The current leads were made of indium, in order to avoid undesirable thermoelectric effects, and were welded directly to the sample ends.⁹ The potential leads were of 0.002-in. Manganin wire attached 3 cm apart at points aligned along the samples axis. The separation between the potential and the current leads was always greater than 0.5 cm. In order to check that the separation between the potential leads did not affect the results, two more wires were attached to sample III separated by a distance of 1.5 cm; the results obtained in this case are identified as sample IV.

The potential leads were point-soldered to the samples. For this, the Manganin wire was slightly flattened in the zone to be soldered. The solderings so

obtained were of good quality, unyielding, and localized within an area about 0.1 mm in diameter. The samples were fixed to a holder which was also made of indium in order to avoid damage due to different thermal contraction. Mylar 0.0075 in. thick, glued with G.E. 7031 varnish, was placed between samples and holder to provide electrical insulation. The samples were afterwards annealed at 50°C for 24 h.

When the samples were at 4.2°K, the spurious emf's were of the order of some microvolts. In order to get rid of this effect, the potential leads were cut away some centimeters from the solderings and continued with copper wire to room temperature. With the cold ends of the copper wires within the helium bath, the spurious emf's were reduced by a factor of about 10.

Method of Measurement

The resistance measurements were made by circulating a constant current through the sample and measuring the voltage drop.

The current was measured using a Weston model 931 ammeter, 3-A full scale, and the voltage was measured with a Keithley model 149 millimicro-voltmeter, 0.1 μ V full-scale maximum sensitivity.

Several current values were used in order to be sure that there was no heating of the samples and that the resistance measurements were current-independent. The correct orientation of the field with respect to the samples was determined using the fact that the resistance must be a minimum when the field is parallel to the current. The magnetic field was measured using a rotating coil gaussmeter. The geometry factor was determined by measuring the resistance of each sample at room temperature and assuming that¹⁰

$$\rho_{293} = 9.1 \times 10^{-6} \Omega \text{ cm.}$$

RESULTS AND DISCUSSION

Bulk Magnetoresistance

Kohler's rule states that the relative variation of the resistance due to the presence of a magnetic field is a function only of the ratio between the magnetic field

TABLE II. Experimental values of the mean free path l , the bulk resistivity ρ_b , and the electron momentum mv obtained from measurements on thin films of various thicknesses a .

Sample	a (cm)	$\rho_b (10^{-10} \Omega \text{ cm})$	$l (10^{-2} \text{ cm})$	$\rho_b l (10^{-11} \Omega \text{ cm}^2)$	$mv (10^{-10} \text{ g cm sec}^{-1})$
V	0.012	1.3	7.25	0.94	1.45
VI	0.0055	2.84	3.47	0.98	1.56
VII	0.0033	2.72	3.31	0.90	1.53
VIII	0.0032	3.62	2.46	0.89	1.49
IX	0.0065	1.76	5.00	0.88	1.55
X	0.011	1.32	6.8	0.90	1.40

⁹ Divco Heavy Duty Flux was used.

¹⁰ Wright Air Development Division Technical Report No. 60-56, Part IV (unpublished).

H and the zero-field resistivity:

$$(\rho_H - \rho) / \rho = F(H/\rho). \quad (1)$$

Here ρ_H is the measured resistivity at field H , and $F(H/\rho)$ is Kohler's function, which depends only on the metal. Kohler's function for indium was determined by measuring the magnetoresistance of the four thickest samples at 4.2°K (see Table I). The results are displayed in Fig. 1. It is interesting to note the good agreement among the data from the different samples. At this temperature, the three samples of lower purity were already in the residual zone, while the pure one was not yet in such a zone. At 1.5°K, since the resistance of the pure sample decreased with temperature, the noise due to the presence of the magnetic field made it impossible to obtain accurate measurements of the magnetoresistance. However, since the results are purity-independent, it is reasonable to assume that they are valid at lower temperatures; we will then use the above-mentioned Kohler's function for analyzing the data at both 4.2 and 1.5°K.

Our results disagree with the results previously obtained by Olsen,⁷ which are plotted, for comparison, in Fig. 1. The reason for this discrepancy is unknown. It was suspected that our results might be affected by the way in which the potential leads were soldered. Accordingly, a sample was made in which the Manganin wire was replaced by indium which was cold-welded to the sample. In this case, the leads were not so tightly fixed and the region affected not so pointlike, but the results were in agreement with the ones previously obtained. As a final check, the magnetoresistance of an aluminum sample was measured, and the results were in good agreement with those obtained by other authors.⁸

Magnetomorphic Effect

The size-effect magnetoresistivity was measured at 4.2 and 1.5°K in six samples made, using material of the same origin as sample I. As previously mentioned, the bulk magnetoresistance was discounted by means of Kohler's rule modified by Olsen:

$$\frac{\rho_H - \rho_{H'}}{\rho_{H'}} = F\left(\frac{H}{\rho_{H'}}\right), \quad (2)$$

where ρ_H is the measured resistivity in the presence

TABLE III. Experimental values of the mean free path l , the bulk resistivity ρ_b , and the electron momentum mv obtained from Olsen's data in thin wires (diameter d) using Kohler's rule of Fig. 1.

d (cm)	ρ_b (10^{-10} Ω cm)	l (10^{-2} cm)	$\rho_b l$ (10^{-11} Ω cm ²)	mv (10^{-19} g cm sec ⁻¹)
0.006	15.0	0.86	1.31	1.39
0.0085	12.1	0.94	1.14	1.32
0.031	8.3	1.82	1.51	1.5

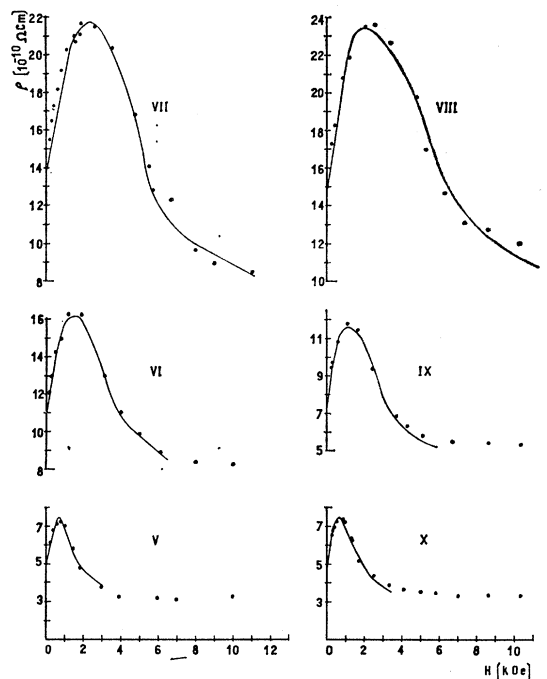


Fig. 2. Resistivity of thin films as a function of magnetic field at 1.5°K. The solid curves give the best adjustment to theory, using parameters shown in Table II. The roman numerals refer to the different samples.

of a field H , $\rho_{H'}$ is considered to be the resistivity that would be measured if no bulk magnetoresistance were present, and $F(H/\rho_{H'})$ is Kohler's function for indium.

At 4.2°K, no reasonable agreement was found between theory and experiment. This disagreement might arise from the fact that at this temperature the samples were not yet in the residual region and the electron-phonon scattering may be important.

At 1.5°K, $\rho_{H'}$ showed a maximum, as predicted by theory, but if the theoretical and experimental curves are fitted at the maximum, there is no agreement at all in the rest of the curve. Nevertheless, it is possible to fit both curves with fairly good agreement in the whole theoretically calculated range at the expense of a somewhat poorer agreement in the region of the maximum. This was the criterion used to compare the results, as shown in Fig. 2.

Actually, in the thickest samples, V and X, no maximum was observed. This is due to the fact that since l and the thickness of the samples, a , are large, the maximum was in the region in which the material is in the superconducting state. Table II shows the values of ρ_b , l , and mv , deduced from the comparison with the theory. Comparing the measured size-effect resistivity with the theoretical curves, the value of mv can be determined within $\pm 7\%$ and, as can be seen in Table II, all the values obtained are within this variation. The mean value of mv is 1.5 g cm/sec.

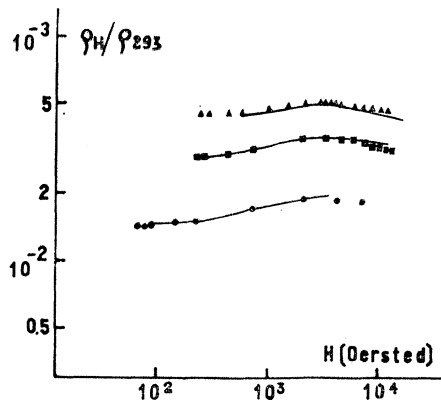


FIG. 3. Olsen's data of resistivity of thin wires as a function of magnetic field at 2°K. The solid curves give the best adjustment to theory using parameters shown in Table III. \blacktriangle , $\phi=0.006$ cm; \blacksquare , $\phi=0.0085$ cm; \bullet , $\phi=0.031$ cm.

Comparison with Other Results

Previous measurements on longitudinal size effects in indium have been carried out by Olsen in thin wires.⁷ He found the agreement between experimental and theoretical curves was poor. However, he made the comparison using a criterion different from the one used in our work. Olsen ascribes the same l to all samples made with the same material. This might not be true because of the difference in contamination of the samples during the manufacturing process, as previously mentioned.

We have fitted Olsen's data by choosing the values of ρ_b , l , and mv that give the best agreement with the theoretical curves, as was done with the thin films. The values of $\rho_b l$ and mv obtained using either of the curves given in Fig. 1 are the same within the accuracy of the method. However, the theoretical curves are best fitted using our measured Kohler's rule, and this

was the one used to calculate the values given in Table III and the curves of Fig. 3.

As can be seen from Table III, the mv values are in good agreement with those obtained by us from the thin-film measurements. The mean value of $\rho_b l$ from the data of Table III is $1.3 \times 10^{-11} \Omega \text{ cm}^2$. This value is a little smaller than the $1.8 \times 10^{-11} \Omega \text{ cm}^2$ obtained by Olsen, assuming the applicability of Nordheim's rule. However, since ρ_b is different for each sample this rule cannot be applied.

The average of $\rho_b l$ from our measurements in thin films is $0.92 \times 10^{-11} \Omega \text{ cm}^2$. This is smaller than the value we obtained from Olsen's data, but still greater than the one considered as the best from measurements of the anomalous skin effect.¹¹ Bate *et al.*¹¹ have investigated the influence of the anisotropy of the mean free path in the measured value of $\rho_b l$ in thin wires. They conclude that because of the anisotropy, the experimental value is greater than the isotropic one. The anisotropy of l is owing to the fact that when the diameter of the wire is comparable to the grain size the sample may be considered as a "one-dimensional polycrystal." In this sense, our samples are "two-dimensional polycrystal," and this may account for the fact that our value for $\rho_b l$ lies between the value obtained from measurements of the longitudinal size effect in indium wires and that from the anomalous skin effect.

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¹¹ R. T. Bate, Byron Martin, and P. F. Mille, Phys. Rev. **131**, 1482 (1963).