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Experimental Results on Pion Production Compared with Predictions of the Isobar Model.

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Summary. – The predictions of the model of Lindenbaum and Sternheimer are compared with results obtained from the analysis of the reactions $\pi^-+p\to\pi^-+\pi^++n$; $\pi^-+p\to\pi^-+\pi^0+p$, observed in an H_2 bubble chamber. It was not possible to find any contradiction and all results concerning momentum spectra and branching ratios are sufficiently well described by the model. In the favorable case: $\pi^-+p\to\pi^++I^-\to\to\pi^++(\pi^-+n)$, the correct Q-value distribution for the isobar is obtained and angular correlations in its decay are compatible with a $J=\frac{3}{2}$ for the (π^-+n)-pair.

1. – Introduction.

The relation between pion-proton scattering and photoproduction phenomena observed at low energy (the « $\frac{3}{2}$-$\frac{3}{2}$ » resonance) suggested that it would be of interest to investigate if also at higher energies the observed resonant state would play an important role, in particular, for pion production. During the last few years a phenomenological model for pion production was developed, in which was brought into consideration the possibility of the presence of an excited state of the nucleon in an intermediate step of the process, namely the $\frac{3}{2}$-$\frac{3}{2}$ resonant state. This excited nucleon should be able to decay into a pion and a nucleon and is well known to be also responsible for the low energy pion-proton resonance.

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The isobar model removes some difficulties appearing when one compares experimental results with earlier models, like Fermi's statistical theory for π-production. Up to now the available experimental material on pion-production in pion-proton and proton-proton collisions was not very extensive (1), but the general agreement with the isobar model was fairly good.

In the present paper we compare the predictions of the detailed isobar model proposed by LINDENBAUM and STERNHEIMER (2) with results obtained from the reactions:

$$(1) \qquad \pi^- + p \to \pi^- + \pi^+ + n$$

$$(2) \qquad \pi^- + p \to \pi^- + \pi^0 + p$$

observed in an hydrogen bubble chamber.

2. – Predictions of the isobar model.

We summarize here some conclusions obtained by STERNHEIMER and LINDENBAUM (3) from their detailed model. According to this model, the reaction:

$$(3) \qquad \pi + \mathcal{N} \to \pi + \pi + \mathcal{N}$$

should take place in two independent steps:

$$(4) \qquad \pi + \mathcal{N} \to \pi + I \to \pi + \pi_{\text{dec}} + \mathcal{N} ,$$

where I stands for « isobar », whose decay products should not interact with the « extra-π » of the first step of the reaction.

There are five reactions of type (4), according to the different possibilities of the electric charge of the isobar and the « extra-π » and the different possibilities of decay of the isobar, namely the following:

$$(5) \qquad \begin{cases} (a) & \pi^- + p \to \pi^- + I^+ \to \pi^- + \pi^+ + n \\ (b) & \pi^- + p \to \pi^- + I^+ \to \pi^- + \pi^0 + p \\ (c) & \pi^- + p \to \pi^+ + I^- \to \pi^+ + \pi^- + n \\ (d) & \pi^- + p \to \pi^0 + I^0 \to \pi^0 + \pi^- + p \\ (e) & \pi^- + p \to \pi^0 + I^0 \to \pi^0 + \pi^0 + n . \end{cases}$$

(1) See reference (3).

(2) S. J. LINDENBAUM and R. B. STERNHEIMER: *Phys. Rev.*, **105**, 1874 (1957).

(3) R. B. STERNHEIMER and S. J. LINDENBAUM: *Phys. Rev.*, **109**, 1723 (1958).

Assuming charge independence it is possible to analyse the above listed reactions in terms of two isotopic spin states ($\frac{1}{2}$ and $\frac{3}{2}$) for the initial system. This analysis is perfomed by STERNHEIMER and LINDENBAUM through two suitable parameters ϱ and α, defined as follows:

$$\varrho = \sigma_{\frac{3}{2}}^{\text{in}}/2\cdot\sigma_{\frac{1}{2}}^{\text{in}}\,, \tag{6}$$

where $\sigma_{\frac{1}{2}}^{\text{in}}$ and $\sigma_{\frac{3}{2}}^{\text{in}}$ are the cross sections for single pion production for isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, and

$$\alpha = 2\cdot(\varrho/5)^{\frac{1}{2}}\cdot\cos\varphi\,, \tag{7}$$

where φ is the phase difference between the matrix elements of isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$. In terms of these parameters there are several relations between the five reactions (a) to (e); in particular the following will be useful for comparison with the experiment:

$$(c)/(a) = \xi_1 = (45 + 36\varrho + 90\alpha)/(5 + 16\varrho - 20\alpha)\,, \tag{8}$$

$$(b)/(d) = \xi_2 = (5 + 16\varrho - 20\alpha)/(5 + \varrho - 5\alpha)\,, \tag{9}$$

$$\big((a) + (c)\big)/\big((b) + (d)\big) = R = (10 + 17\varrho - 25\alpha)/(25 + 26\varrho + 35\alpha)\,, \tag{10}$$

$$(e)/\big((a) + (b) + (c) + (d) + (e)\big) = S = (10 + 2\varrho - 10\alpha)/45\,(1 + \varrho). \tag{11}$$

Now, if the numerical values of ξ_1, ξ_2, R, and S are fixed (from the experiment, for instance) then equations (8) to (11) represent linear relations between ϱ and α; ϱ should simultaneously satisfy (6). This last condition contains the inelastic $\pi^+ + p$ cross section through the relations:

$$\sigma_{\pi^+}^{\text{in}} = \sigma_{\frac{3}{2}}^{\text{in}} \qquad \sigma_{\pi^-}^{\text{in}} = \tfrac{2}{3}\,\sigma_{\frac{1}{2}}^{\text{in}} + \tfrac{1}{3}\,\sigma_{\frac{3}{2}}^{\text{in}}\,. \tag{12}$$

Clearly all these equations are superabundant and this situation provides an interesting check of the reliability of the model.

Furthermore the isobar model allows one to calculate the spectra of the emitted particles if several reasonable assumptions are made, for instance, if it is assumed that the probability to form a $\frac{3}{2}$-$\frac{3}{2}$ isobar of a certain mass is related to the $\pi^+ + p$ scattering cross section at the corresponding energy [3] (phase space factors included). Fig. 1 shows the spectra for the « extra-π » and for the « decay-π » calculated by F. SELLERI [4] for an incident pion K.E. of 960 MeV.

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[4] F. SELLERI: *Thesis* (University of Bologna), unpublished.

The peak in the « extra-π » spectrum reproduces, apart from the phase space factors, the peak in the $\pi^+ + p$ cross section.

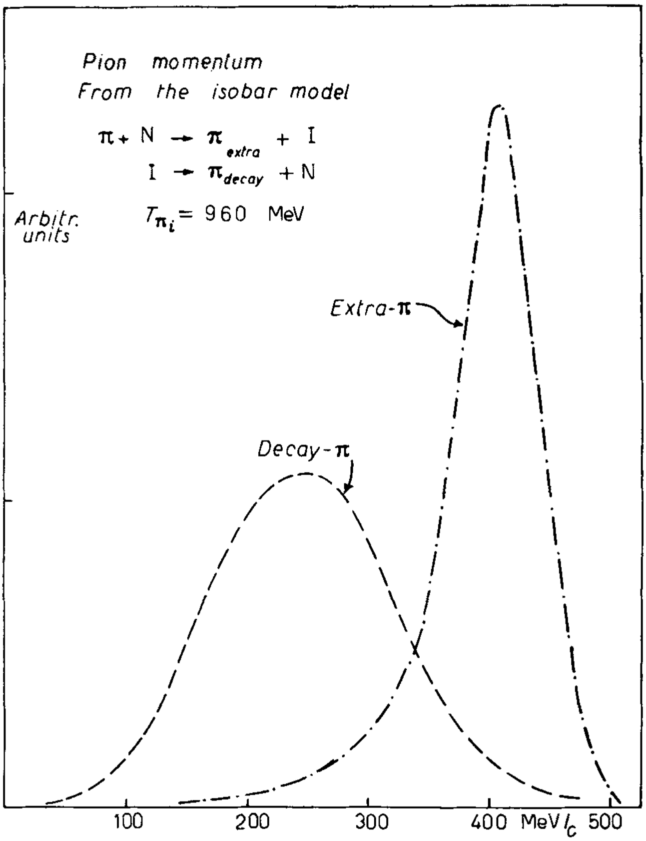

Fig. 1.

The experimentally observed reaction (1) is a superposition of reactions (a) and (c) of (5). In favorable conditions it will be possible to deduce the branching ratio $(c)/(a) = \xi_1$ from the spectrum of the emitted pions. But it is also possible to eliminate ξ_1 lumping together the spectra of the pions of both signs. The resulting spectrum is independent of ϱ and α. In the same way the momentum spectra of the pions from reaction (2) will give information on the parameter ξ_2 and again the spectrum resulting from adding both pions is independent of ϱ and α.

Within the framework of the isobar model the correlation between production and decay angles could give a possibility to determine the spin of the « isobar » in a similar way as ADAIR (5) proposed for the Λ^0-hyperon in the process $\pi^- + p \rightarrow \Lambda^0 + \theta^0$. The spin of the « second » particle emitted is here well known and the argument can be applied as was also observed by MORPURGO (6). One expects a distribution: $1+3\cos^2\theta$ for the decay products of the $\frac{3}{2}$, $\frac{3}{2}$-isobar. The angle of decay must be measured with respect to the line of flight of the incident particles but in the CMS of the decaying isobar (7). In the case in which the isobar is produced not merely in s-waves, only events produced backwards and forwards should be used.

In its present state the isobar model gives no predictions on the absolute values of the cross-sections, except under particular assumptions as those introduced by LINDENBAUM and STERNHEIMER (8). The problem is related to the states of angular momentum involved in the production process. Some arguments in this sense could perhaps be obtained from the analysis of the angular distribution for isobar production.

3. – Experimental results.

The events here reported were obtained in a new scan of pictures of the hydrogen bubble chamber of the Columbia University, exposed to the 960 MeV pion beam of the Brookhaven Cosmotron. The same film was previously used

TABLE I (*).

Clear elastic events	521	(417)	
Stops	222	(190)	
Clear $\pi^0+\pi^-+p$	190		(355)
Clear $\pi^-+\pi^++n$	240		
« n » or « p »	13		
Clear $\pi^-+\pi^-+\pi^0+n$	43		
Probable $\pi^-+\pi^0+\pi^0+p(+?)$	8		
Clear $\pi^-+\pi^-+\pi^++p$	23	(21)	
Strange particles	25		
Total	1285	(983)	

(5) R. K. ADAIR: *Phys. Rev.*, **100**, 1540 (1955).

(6) G. MORPURGO: *Nuovo Cimento*, **9**, 564 (1958).

(7) F. EISLER, R. PLANO, A. PRODELL, N. SAMIOS, M. SCHWARTZ, J. STEINBERGER, P. BASSI, V. BORELLI, G. PUPPI, G. TANAKA, P. WALOSCHEK, V. ZOBOLI, M. CONVERSI, P. FRANZINI, I. MANNELLI, R. SANTANGELO, V. SILVESTRINI, G. L. BROWN, D. A. GLASER and C. GRAVES: *Nuovo Cimento*, **7**, 222 (1958).

(8) S. J. LINDENBAUM and R. B. STERNHEIMER: *Phys. Rev.*, **106**, 1107 (1957).

(*) Data between brackets are those obtained by ERWIN and KOPP (9) in a similar experiment. They are in good agreement with our numbers.

(9) A. R. ERWIN and J. K. KOPP: *Phys. Rev.*, **109**, 1364 (1958).

for strange particles experiments ([10]). All events observed in a well defined region of the chamber were analysed. Of a total of 1285 interactions observed, there were found 240 of reaction (1) (neutrons) and 190 of reaction (2) (protons). The classification of all events is shown in Table I. Favorable conditions of the chamber allowed a clear identification of nearly all events. The measured momenta were first adjusted with a least squares fit (in order to balance momentum and energy) and then transformed into the CMS by means of an IBM-650 computer. Details of the measurements and computations as well as some results not included in this paper will be published separately.

Reaction (1) is the most favorable case to observe the isobar. The (π^-+n)-pair is a pure isotopic spin $\frac{3}{2}$ state, and the isobar model predicts that reaction (1)

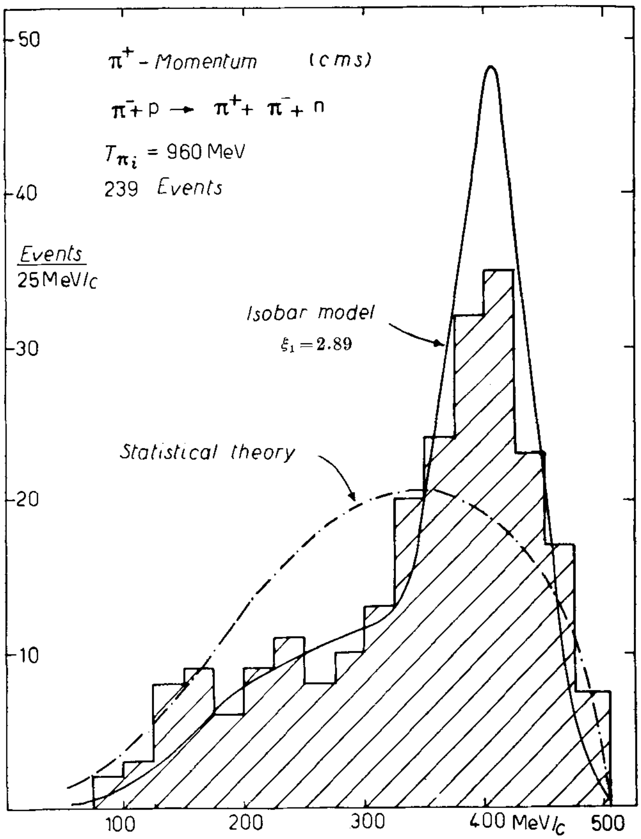

Fig. 2.

([10]) F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, H. Tanaka, P. Waloschek, V. Zoboli, M. Conversi, P. Franzini, I. Manelli, R. Santangelo and V. Silvestrini: *Nuovo Cimento*, **10**, 468 (1958).

goes mainly through the channel (*c*). The momentum spectrum of the positive pions is shown in Fig. 2 together with the spectrum obtained from the isobar model. The values of ϱ and α used here will be justified later. The peak at 400 MeV corresponds to the $\frac{3}{2}$, $\frac{3}{2}$ resonance. The π^--spectrum reflects the same situation and shows the small effect of the $(\pi^+ + p)$-isobar (Fig. 3).

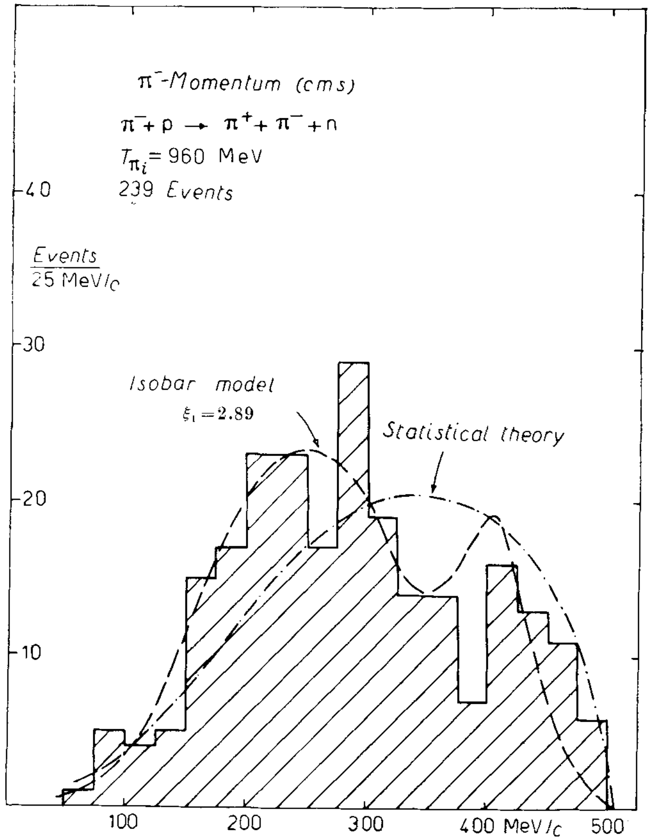

Fig. 3.

Figs. 4 and 5 show the results from reaction (2) and are compared with the spectra obtained from the isobar model, using the same values of ϱ and α as before (full line). The dotted curves, which correspond to values of ξ_2 determined from each spectrum as will be shown later, give a better fit.

As it was remarked before, the superposition of the two spectra of Fig. 2 and 3 is independent of the initial mixture of isotopic spin states. The same holds for Figs. 4 and 5 and also for the sum of all pion spectra. This latter is shown in Fig. 6 and it is in agreement with the spectrum predicted by the isobar model.

In order to get the branching ratios defined in (8) and (9) we first divided each experimental spectrum into two regions: region « A » over 325 MeV/c and region « B » under 325 MeV/c. Region « A » contains 90% of the extra pion

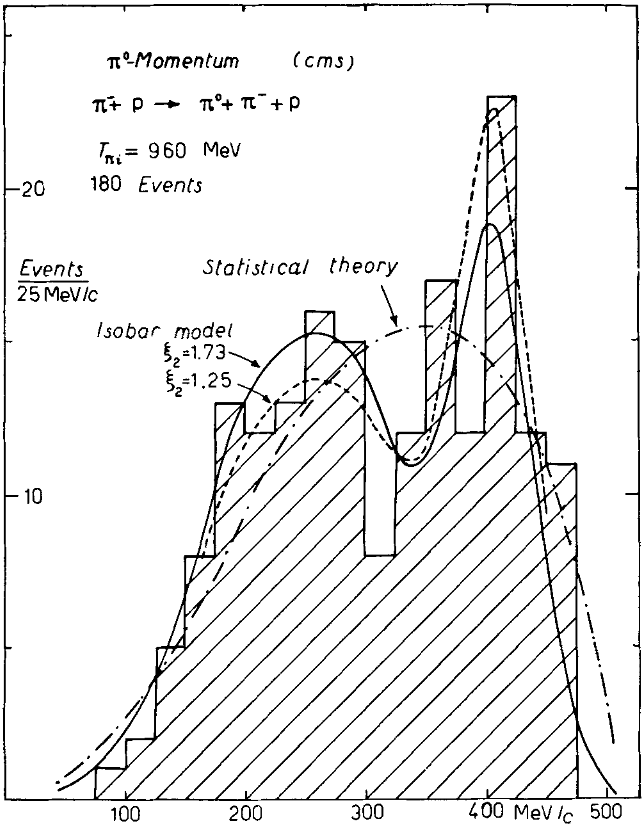

Fig. 4.

spectrum and 15% of the decay pion spectrum, while region « B » contains 85% of the decay pion spectrum and 10% of the extra pion spectrum. These percentages were obtained from the theoretical spectra of Fig. 1. Now, from the number of events observed in « A » and « B » we can deduce the ratio « extra-π »/« decay-π » for every spectrum. This procedure seems more reliable to us than a least squares fit, since small measurement errors could displace some events (in particular from the narrow peak) slightly deforming the spectrum, but the sum of all events contained in « A » and « B » could compensate in part this effect as well as the influence of an eventual anisotropy of the decay angular distribution of the isobar which would only affect the form of the decay pion part.

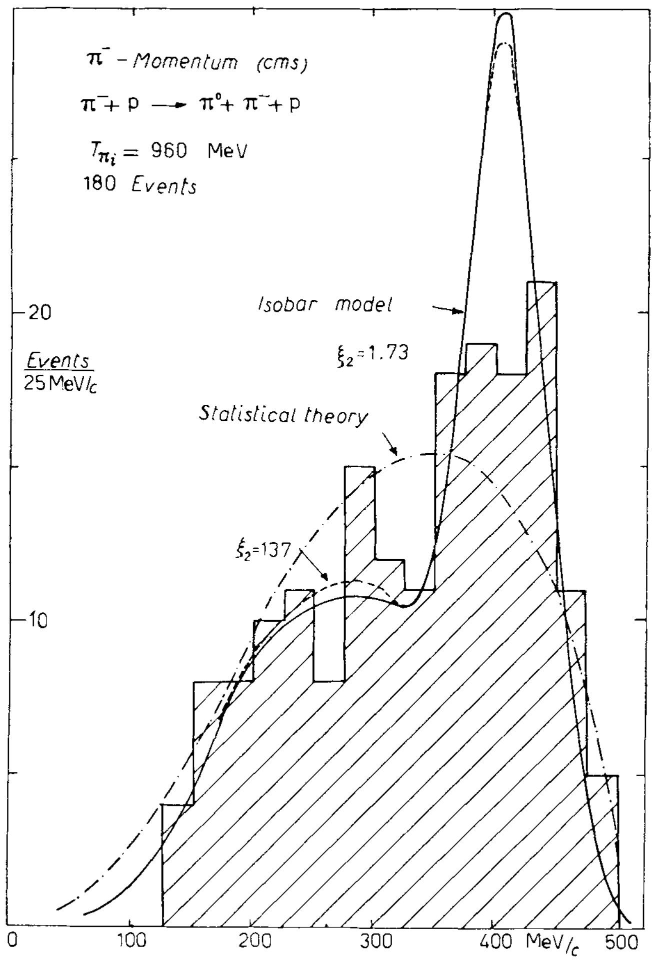

Fig. 5.

All pion spectra were treated in an identical way, and we obtained:

$\xi_1 = 2.25 \pm 0.43$ from the π^+-spectrum (n)

$\xi_1 = 2.98 \pm 0.60$ from the π^--spectrum (n)

$\xi_2 = 1.32 \pm 0.27$ from the π^--spectrum (p)

$\xi_2 = 1.25 \pm 0.25$ from the π^0-spectrum (p).

The branching ratio R is obtained from Table I:

$$R = 0.792 \pm 0.077 = \text{protons/neutrons}\,.$$

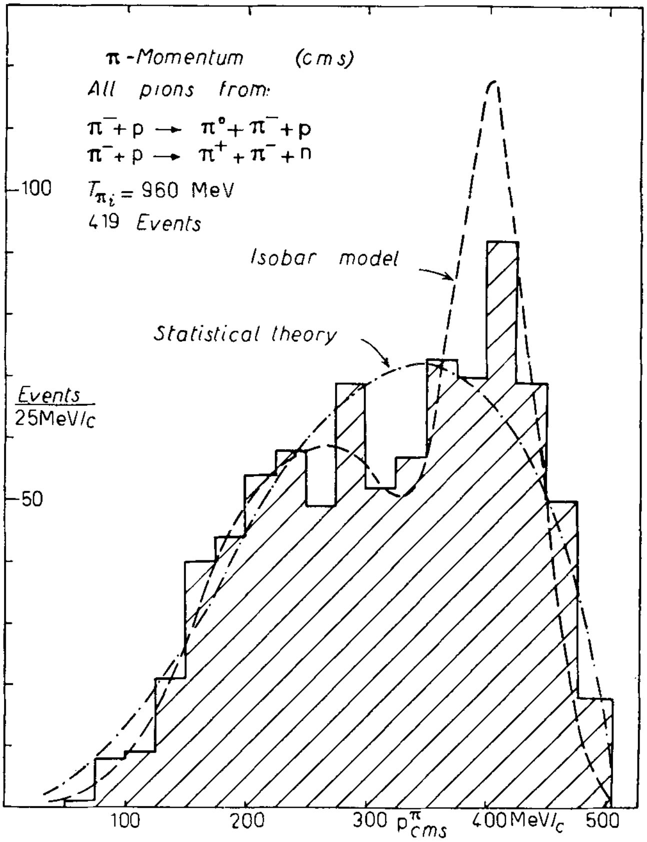

Fig. 6.

The number of « stops » gives an upper limit for the reaction (e) since this number contains also the normal charge exchange events. We obtain:

$$S < 0.334 \pm 0.026 = \text{stops/total}\,.$$

Before discussing the compatibility between all these numbers, we shall show the results of the analysis of the angular correlations.

We first divided all events into four classes, corresponding to reactions (a), (b), (c) and (d). As « extra-π » the more energetic one was always selected. The samples so obtained are contaminated with wrongly classified events in

~ 30% for class (*a*), ~ 10% for class (*b*), ~ 5% for class (*c*) and ~ 20% for class (*d*). Figs. 7 to 9 show the angular distribution of the production of the differently charged isobars, without any correction for their mutual contamination. It is clear from these distributions that different waves are involved in each case.

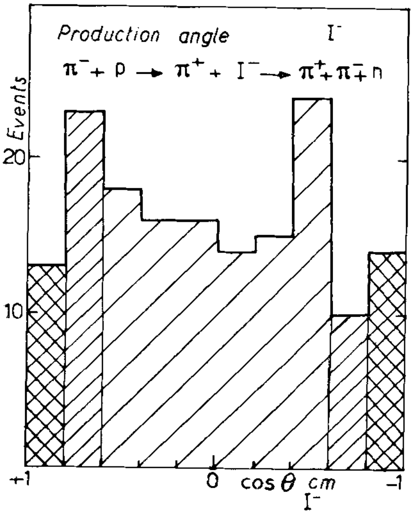

Fig. 7.

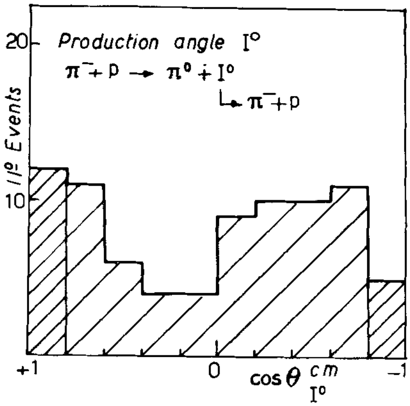

Fig. 8.

For the spin analysis we first considered reaction (*c*). As the production is roughly isotropic, a first attempt would be to assume that the isobar is emitted with $l = 0$. Thus, all events could be used for Adair's analysis, as is shown in Fig. 10. A curve $(1 + 3\cos^2\theta)$ normalized to the total number of events has been superimposed. The agreement is not too good, but it becomes better by selecting events with a small production angle. Corresponding regions are equally shadowed on the decay angle distribution and on the production angle distribution ($|\cos\alpha_{pr}| > 0.6$ and > 0.8). The result is compatible with the assumed $J = \frac{3}{2}$ of the isobar, even though the statistics is not sufficient for a spin determination.

The angular distributions for the production of the other charge states of the isobar are not so simple and it is possible that the limit in the production angle for a successful spin analysis is very small.

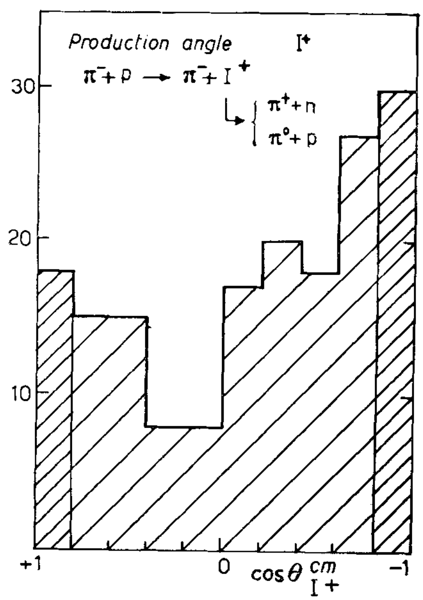

Fig. 9.

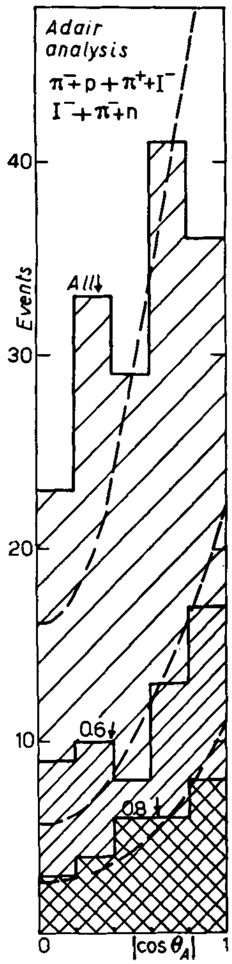

Fig. 10.

Fig. 11 shows the decay distributions for all events and for those produced at $|\cos\alpha_{pr}| > 0.8$. With the present statistics it is not possible to reduce any further the limit for the production angle and it remains doubtful if any conclusions could be obtained from this last analysis of reactions (a), (b) and (d).

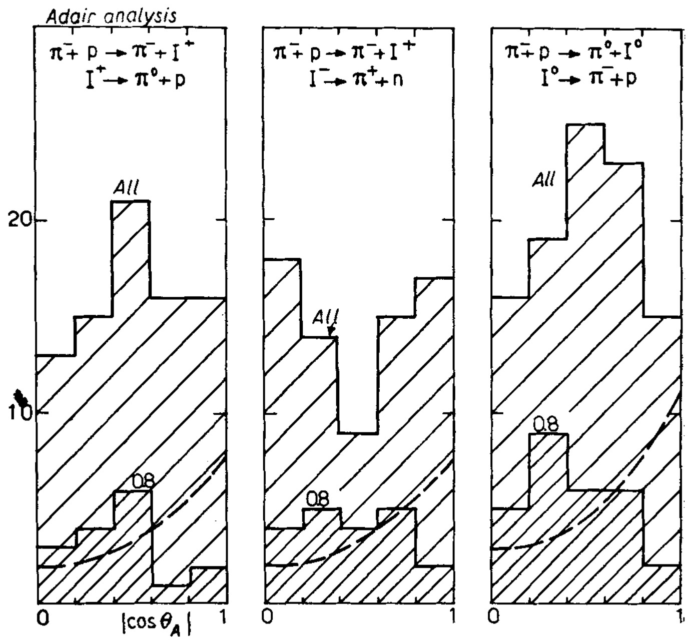

Fig. 11.

4. – Discussion.

As we have shown, momentum spectra and angular correlations seem not to contradict the isobar model. We should now analyse the situation with respect to the branching ratios defined in (8) to (11). As we already said the parameters ϱ and α are related through linear equations containing ξ_1, ξ_2, R S, $\sigma^{\text{in}}_{\frac{1}{2}}$ and $\sigma^{\text{in}}_{\frac{3}{2}}$. The experimental results on R and ξ_1, define α as very close to -0.2. ϱ could instead vary between 0.02 and 0.2, depending essentially on the value of $\sigma^{\text{in}}_{\pi^+}$ (*). We have taken $\sigma^{\text{in}}_{\pi^+} = 6.3$ mb and consequently

$$\xi_1 = 2.89, \qquad \xi_2 = 1.73, \qquad \varrho = 0.1\,, \qquad \alpha = -0.2\,,$$

(*) $\sigma^{\text{in}}_{\pi^+}$ is understood as referring to production of only one pion.

as convenient values for the isobar model calculations. For this compromise we had to go quite far from the experimentally determined value of ξ_2 but, as it was shown on the pion spectra (Figs. 4 and 5), this does not seem to be in contradiction with the experimental data (*).

The number of « stops » corresponding to the reaction (e) obtained from our ϱ and $\varkappa$ is 140. Our experimental number of 222 should then contain 82 charge exchange events. Since we have simultaneously fixed $\sigma^{el}_{\pi^+}$ ($=\sigma^{tot}_{\pi^+}-\sigma^{in}_{\pi^+}$) one can verify that there is no contradiction in the « triangular relations » between $\sqrt{\sigma^{el}_{\pi^-}}$, $\sqrt{\sigma^{el}_{\pi^+}}$ and $\sqrt{2\sigma_{ch.ex}}$.

The whole situation does not change very much if we assume any value of $\sigma^{in}_{\pi^+}$ up to about 10 mb (*i.e.* taking $\varrho<0.18$). Only if $\sigma^{in}_{\pi^+}$ turned out to be higher than this upper limit we would find some trouble in fitting the momentum spectra of the reaction (2) ($\pi^-+\pi^0+p$).

5. – Conclusions.

It was not possible for us to find any contradiction between our experimental results and the very precise predictions of the isobar model of Sternheimer and Lindenbaum, since the model describes sufficiently well all the results concerning momentum spectra and branching ratios worked out from the present statistics. With respect to angular correlations, only for reaction (c) a reasonable number of events is available; although not statistically significant for a spin determination, it gives distributions which are compatible with the assumed spin $\frac{3}{2}$ for the isobar.

Results from π^++p reactions will provide a more precise check of the validity of the model.

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We are very grateful to the bubble chamber group of the Columbia University, in particular to Prof. J. STEINBERGER for allowing us to perform the measurements here presented on photographs of their chamber, obtained at the Cosmotron.

The continuous encouragement and interest as well as the valuable suggestions of Prof. G. PUPPI, which made possible this work, are gratefully acknowledged.

In the first stage of the measurements Dr. A. MINGUZZI-RANZI and Dr. V. ZOBOLI collaborated with us. For many helpful discussions we thank Prof. A. MINGUZZI, Dr. F. SELLERI and Dr. L. BERTOCCHI.

(*) To fit ξ_2 within one standard deviation and simultaneously ξ_1 and R, ϱ should be taken near 0.05, $\sigma^{in}_{\pi^+}\approx 3$ mb.

The help of Miss A. DIAZ ROMERO and Mr. C. ALVISI in the computation work is gratefully recognized.

Most of the calculations were performed at the IBM-650 of the Centro Calcoli of the University of Bologna. We are indebted to the members of its staff, in particular to Dr. Ing. F. PIERANTONI and Dr. A. CHIARINI who gave us helpful advices for the programming work.

One of us (E.P.F.) wishes to thank Prof. G. PUPPI and the staff of the « Istituto di Fisica A. Righi » in Bologna, for the kind hospitality extended to her; she is also indebted to the Comisión Nacional de Energía Atómica, Rep. Argentina, for a fellowship.

RIASSUNTO

Si confrontano le previsioni del modello di Lindenbaum e Sternheimer con i risultati ottenuti dall'analisi delle reazioni $\pi^-+p\to\pi^-+\pi^++n$; $\pi^-+p\to\pi^-+\pi^0+p$, osservate in una camera a bolle ad idrogeno. I risultati sperimentali non sono in contraddizione con il modello e sono da esso descritti sufficientemente bene. Nel caso favorevole $\pi^-+p\to\pi^++I^-\to\pi^++(\pi^-+n)$ il modello dà la distribuzione corretta dei valori di Q e le correlazioni angolari nel decadimento sono compatibili con un $J=\frac{3}{2}$ per le coppie (π^-+n).