

Therefore,

$$B_{T_1} - B_{T_2} = \frac{(\gamma\delta)}{V_0} \left\{ T_2 \left[\frac{E(\Theta/T)}{T} \right]_{T=T_2} - T_1 \left[\frac{E(\Theta/T)}{T} \right]_{T=T_1} \right\}.$$

The value of $(\gamma\delta)$ can now be evaluated from the experimental values of B_s at any two temperatures, and, in turn, is used to calculate the bulk moduli at various temperatures with a proper arbitrary parameter B_0 . A comparison of the estimated and experimental values are given in Fig. 3.

In all the cases examined, a good agreement is found between the calculated and experimental values; and, therefore, Wachtman's equation, as modified by Anderson, can very well be used for estimating the bulk moduli even at temperatures at which the experiments have not been carried out. Generally, it is observed that the estimation is quite near to the experimental points when the room temperature value of γ and $\delta = 2\gamma$ are used. We thus confirm our earlier conclusion[1] that the Wachtman's equation can safely be used for calculating the temperature dependence of bulk modulus in oxide and nonoxide solids.

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Magnon-phonon interaction in the itinerant electron model of ferromagnetism

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WE WANT to present here a derivation of the bilinear magnon-phonon coupling using as a starting point the electron-phonon coupling and the itinerant electron model of ferromagnetism. The consequences of this coupling are well known and have been discussed by Kittel[1] from a phenomenological point of view.

The model Hamiltonian for a metallic ferromagnet in the presence of a magnetic field H is[2]

$$H_{el} = \sum_{k\sigma} \epsilon_{k\sigma} C_{k\sigma}^+ C_{k\sigma} + \frac{U}{N} \sum_{kk'q} C_{k+q\uparrow}^+ C_{k\uparrow} C_{k'-q\downarrow}^+ C_{k'\downarrow} \quad (1)$$

with

$$\epsilon_{k\sigma} = \epsilon_k - \sigma\mu H \quad (2)$$

where ϵ_k is the energy of the Bloch state $|k\sigma\rangle$ and μ is the Bohr magneton. $C_{k\sigma}^+$ is the creation operator of an electron in the state $|k\sigma\rangle$. U is the Coulomb integral, $\sigma = (\uparrow, \downarrow)$.

Following Rajagopal and Joshi[3] we take the electron-phonon interaction in the form

$$H_{el-ph} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}\mathbf{q}\lambda} \sum_{\sigma\sigma'} g_{\sigma\sigma'}^{\lambda}(\mathbf{q}) C_{\mathbf{k}+\mathbf{q}\sigma}^{\dagger} C_{\mathbf{k}\sigma'} b_{\mathbf{q}\lambda} + h.c. \quad (3)$$

The parameter $g_{\sigma\sigma'}^{\lambda}(\mathbf{q})$ measures the strength of the coupling. $b_{\mathbf{q}\lambda}$ is the destruction operator for a phonon of momentum \mathbf{q} and polarization λ . From this interaction Hamiltonian we consider only those terms which give direct magnon-phonon coupling, i.e. we shall keep as interaction Hamiltonian

$$H_{\text{max-ph}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}\mathbf{q}\lambda} g_{\uparrow\downarrow}^{\lambda}(\mathbf{q}) B_{\mathbf{k}\mathbf{q}}^{\dagger} b_{\mathbf{q}} + h.c. \quad (4)$$

where

$$B_{\mathbf{k}\mathbf{q}}^{\dagger} = C_{\mathbf{k}-\mathbf{q}\downarrow}^{\dagger} C_{\mathbf{k}\uparrow}$$

Furthermore, since we shall be interested only in the qualitative aspects of the interaction we shall drop in what follows the polarization index λ . For an isotropic model, angular momentum conservation implies that only circularly polarized phonons would be coupled to the magnons. The total Hamiltonian of the system is then:

$$H = H_{el} + H_{ph} + H_{\text{max-ph}} \quad (5)$$

where

$$H_{ph} = \sum_{\mathbf{q}} \omega_{\mathbf{q}}^{\text{PH}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}$$

is the free phonon Hamiltonian.

Following the equation of motion method [4] we try now to find simultaneous eigenfrequencies within the random-phase approximation (RPA) corresponding to the excitation created by $b_{\mathbf{q}}^{\dagger}$ (phonons) and

$$B_{\mathbf{q}}^{\dagger} = \sum_{\mathbf{k}} \alpha_{\mathbf{k}} B_{\mathbf{k}\mathbf{q}}^{\dagger} \text{ (magnons)}$$

$\alpha_{\mathbf{k}}$ can be determined later, in solving for the eigenfrequencies. After a simple calculation we find:

$$\begin{aligned} iB_{\mathbf{k}\mathbf{q}}^{\dagger} &= [H, B_{\mathbf{k}\mathbf{q}}^{\dagger}] = (\epsilon_{\mathbf{k}+\mathbf{q}\downarrow} - \epsilon_{\mathbf{k}\uparrow}) B_{\mathbf{k}\mathbf{q}}^{\dagger} \\ &\quad - \frac{U}{N} \sum_{\mathbf{k}'\mathbf{q}'} C_{\mathbf{k}+\mathbf{q}\downarrow}^{\dagger} C_{\mathbf{k}-\mathbf{q}'\uparrow} C_{\mathbf{k}'-\mathbf{q}'\downarrow}^{\dagger} C_{\mathbf{k}'\uparrow} \\ &\quad - \frac{U}{N} \sum_{\mathbf{k}'\mathbf{q}'} C_{\mathbf{k}\uparrow} C_{\mathbf{k}'+\mathbf{q}'\downarrow}^{\dagger} C_{\mathbf{k}'\uparrow} C_{\mathbf{k}+\mathbf{q}-\mathbf{q}'\downarrow}^{\dagger} \\ &\quad + \frac{1}{\sqrt{N}} \sum_{\mathbf{q}'} g^{*}(\mathbf{q}') (C_{\mathbf{k}+\mathbf{q}-\mathbf{q}'\downarrow}^{\dagger} C_{\mathbf{k}\uparrow} \\ &\quad - C_{\mathbf{k}-\mathbf{q}\downarrow}^{\dagger} C_{\mathbf{k}'-\mathbf{q}'\downarrow}) b_{\mathbf{q}}^{\dagger} \end{aligned} \quad (6)$$

and

$$ib_{\mathbf{q}}^{\dagger} = [H, b_{\mathbf{q}}^{\dagger}] = \omega_{\mathbf{q}}^{\text{PH}} b_{\mathbf{q}}^{\dagger} + \frac{g(\mathbf{q})}{\sqrt{N}} \sum_{\mathbf{k}} B_{\mathbf{k}\mathbf{q}}^{\dagger} \quad (7)$$

within the RPA equation (6) becomes:

$$\begin{aligned} iB_{\mathbf{k}\mathbf{q}}^{\dagger} &= (E_{\mathbf{k}+\mathbf{q}\downarrow} - E_{\mathbf{k}\uparrow}) B_{\mathbf{k}\mathbf{q}}^{\dagger} \\ &\quad + \frac{U}{N} (n_{\mathbf{k}+\mathbf{q}\downarrow} - n_{\mathbf{k}\uparrow}) \sum_{\mathbf{k}'} B_{\mathbf{k}'\mathbf{q}}^{\dagger} \\ &\quad + \frac{g^{*}(\mathbf{q})}{\sqrt{N}} (n_{\mathbf{k}\uparrow} - n_{\mathbf{k}-\mathbf{q}\downarrow}) b_{\mathbf{q}}^{\dagger} \end{aligned} \quad (8)$$

where

$$E_{\mathbf{k}\sigma} = \epsilon_{\mathbf{k}\sigma} + \frac{U}{N} n_{-\sigma}$$

with

$$n_{\sigma} = \sum_{\mathbf{k}} n_{\mathbf{k}\sigma} = \sum_{\mathbf{k}} \langle C_{\mathbf{k}\sigma}^{\dagger} C_{\mathbf{k}\sigma} \rangle.$$

Fourier transforming equations (7) and (8) and eliminating $b_{\mathbf{q}}^{\dagger}$ we obtain

$$\begin{aligned} \Omega B_{\mathbf{k}\mathbf{q}}^{\dagger} &= (E_{\mathbf{k}+\mathbf{q}\downarrow} - E_{\mathbf{k}\uparrow}) B_{\mathbf{k}\mathbf{q}}^{\dagger} + \frac{1}{N} (n_{\mathbf{k}+\mathbf{q}\downarrow} - n_{\mathbf{k}\uparrow}) \\ &\quad \left[U - \frac{|g(\mathbf{q})|^2}{\Omega - \omega_{\mathbf{q}}^{\text{PH}}} \right] \sum_{\mathbf{k}'} B_{\mathbf{k}'\mathbf{q}}^{\dagger} \end{aligned} \quad (9)$$

for the Fourier amplitudes $B_{\mathbf{k}\mathbf{q}}^{\dagger}(\Omega)$.

Multiplying equation (9) by $\alpha_{\mathbf{k}}$ and summing over \mathbf{k} we are led to a set of homogeneous equations involving the $B_{\mathbf{k}\mathbf{q}}^{\dagger}$. The independence of the $B_{\mathbf{k}\mathbf{q}}^{\dagger}$ implies a new set of equations for the $\alpha_{\mathbf{k}}$ from which we obtain the following

eigenvalue equations[4]

$$\frac{1}{N} \sum_{\mathbf{k}} \frac{1}{\Omega - E_{\mathbf{k}+\mathbf{q}\downarrow} - E_{\mathbf{k}\uparrow}} (n_{\mathbf{k}+\mathbf{q}\downarrow} - n_{\mathbf{k}\uparrow}) \left[U - \frac{|g(\mathbf{q})|^2}{\Omega - \omega_{\mathbf{q}}^{PH}} \right] = 1. \quad (10)$$

Assuming a spherical Fermi surface for evaluating the sum in equation (10) we obtain to second order in \mathbf{q}

$$\left[U - \frac{|g(\mathbf{q})|^2}{\Omega - \omega_{\mathbf{q}}^{PH}} \right] \left\{ \frac{n_{\downarrow}}{\Delta} \left[1 - \frac{\epsilon_{\mathbf{q}}}{\Delta} \left(1 - \frac{4\epsilon_{F\downarrow}}{5\Delta} \right) \right] - \frac{n_{\uparrow}}{\Delta} \left[1 + \frac{\epsilon_{\mathbf{q}}}{\Delta} \left(1 + \frac{4\epsilon_{F\uparrow}}{5\Delta} \right) \right] \right\} = 1 \quad (11)$$

where $\Delta = \Omega - Um - 2\mu H$, $m = \sum_{\mathbf{k}} (n_{\mathbf{k}\uparrow} - n_{\mathbf{k}\downarrow})$ is the net moment, and $\epsilon_{F\sigma}$ is the Fermi energy for the up and down bands respectively. Taking now $\mu H \ll Um$ and using the magnon frequency

$$\omega_{\mathbf{q}}^M = 2\mu H + \epsilon_{\mathbf{q}} \left[\frac{n}{m} - \frac{4}{5} \frac{1}{Um^2} (\epsilon_{F\uparrow} n_{\uparrow} - \epsilon_{F\downarrow} n_{\downarrow}) \right]$$

the roots of equation (11) can be written as:

$$\Omega = \frac{\omega_{\mathbf{q}}^{PH} + \omega_{\mathbf{q}}^M}{2} \pm \left[\left(\frac{\omega_{\mathbf{q}}^{PH} - \omega_{\mathbf{q}}^M}{2} \right)^2 + |g(\mathbf{q})|^2 m \right]^{1/2}. \quad (12)$$

We see from (12) that the gap at resonance

$$\Delta\Omega = |g(\mathbf{q})| m^{1/2}$$

is directly related to the electron-phonon coupling.

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Field emission in high magnetic fields

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THE BEHAVIOR of a field emission current in a magnetic field has been of interest for some time [1, 2]. Blatt [3] was the first to theoretically investigate such behavior for a gas of free electrons at the absolute zero of temperature. His results showed that the current should exhibit oscillations which are periodic in H^{-1} . Independently, and very shortly after Blatt's calculation, Gogadze, Itskovich, and Kulik [4] considered the more general case of an arbitrary Fermi surface and non-zero temperature, with the restriction that the electron orbits around the extremal areas are closed and non-self-intersecting. Both papers consider the case for which the Fermi energy is independent of magnetic field, and for which the Fermi energy oscillates with magnetic field. Gogadze *et al.* [4] estimated that observable oscillations in both cases are a possibility only when the current originates from small pockets of electrons with low effective masses. This was essentially the same conclusion reached by Blatt. Numerical estimates of the amplitude of the oscillations from both calculations indicate effects on the order of 2 or 3 per cent at 1 tesla for an electron pocket with an effective mass of $\sim 10^{-2} m_0$. At 3 teslas the amplitudes should be nearly 10 per cent of the emission current in zero magnetic field.

In addition to the oscillatory terms, Blatt [3]